103 學年度第 2 學期應用經濟與管理學系微積分二期末考 Date：2015.06.29(一)09：10-11：00

班級：　　學號：　　姓名：

說明：1.請將每題的題號和計算過程詳細書寫於答案紙上，無須完整抄題。2.滿分 110 分。

1.(18%)求極大(小)值，並請判斷極大或是極小：

(1) $f(x,y)=2x^4+y^2-12xy$

(2) $f(x_1,x_2)=x_1^2x_2-4x_1^2-x_2^2$.

(3) ~~$f(x_1,x_2)=e^{-(x_1^2+x_2^2)}$ s.t. $2x_1+3x_2=4$.~~ $f(x,y)=8x^2y$. s.t. $3x-y=9$

2.(15%)判斷函數是為(半)正定式、(半)負定式或是不定式.

(1) $f(x,y,z)=x^2+6y^2+3z^2-2xy-4yz$.

(2) $f(x,y)=x^2+y^2$.

(3) ~~$f(x,y)=xy$~~ $f(x,y,z)=xy$

3.(5%)Solve the differential equation：$\frac{dy}{dx}+y=4e^x$, $y(0)=50$

4.(5%)求 $\int_0^3[\int_1^2(xy^3-x+4y)dy]dx$

5.(5%)求 $\iint_R(5x+8y)dxdy$ ；$1\le x\le 3$，$0\le y\le x-1$

6.(5%)請利用全微分法，推估 $\sqrt[3]{9.98\times(10.2)^2}$ 用四捨五入法計算到小數後第4位

7.(5%)若 $f(x)=kx^{\frac{3}{2}}$ 為一機率密度函數，$x\in[4,9]$，求 k.

8.(12%) $f(x)=\begin{cases}\frac{x^3}{12}, & 0\le x\le 2\\ \frac{16}{3x^3}, & x>2\end{cases}$ 求① $P(0\le X\le 2)$ ② $P(1\le X\le 3)$

9.(15%)若 $f(x)=0.01e^{-0.01x}$ 為一機率密度函數，$x\in[0,\infty)$，請算出：(1)平均數.(2)變異數.(3)比平均數大一個標準差和比平均數小一個標準差的機率. 求 $f(x)=ae^{-ax}$ 為一PDF下的 μ, σ^2.

10. (10%)某公司的成本函數為 $C(x,y)=2200+27x^3-72xy+8y^2$，請計算出該公司生產這兩項產品下的最小成本與生產數量分別是多少？~~請寫出此時的成本為最小的判別條件.~~

11. (5%)Solve the function of maximizing $f(x,y)=xy$，subject to the constraint

$xy+20y+20x+474{,}000=500{,}000$

12. (10%)If $f(x)=\sqrt{x+1}$，

(1)find the Taylor polynomial of degree 4 at $x=0$.

(2)請利用上面的結果推估 $\sqrt{0.9}$ (僅須列出式子即可)

祝：大家暑假平安、充實！

1032 微積分期末考解答

1. (18%)

(1) $f(x,y)=2x^4+y^2-12xy$

F.O.C.:

$$\begin{cases} f_x = 8x^3-12y = 4(2x^3-3y)=0 \\ f_y = 2y-12x = 2(y-6x)=0 \end{cases}$$

$y=6x$ 代入 $f_x = 4(2x^3-3y)=4(2x^3-18x)$

$= 8(x^3-9x)=0$

$8x(x^2-9)=0$

$x=0$	$y=0$	$\lvert H\rvert = 48(0-3)<0$.
$x=3$	$y=18$	$\lvert H\rvert = 48(9-3)>0$. Min $=f(3,18)=-162$
$x=-3$	$y=-18$	$\lvert H\rvert = 48(9-3)>0$. Min $=f(3,18)=-162$

S.O.C.:

$$|H| = \begin{vmatrix} 24x^2 & -12 \\ -12 & 2 \end{vmatrix} = 48x^2-144 = 48(x^2-3)$$

(2) $f(x_1,x_2)=x_1^2x_2-4x_1^2-x_2^2$

F.O.C.

$f_{x_1} = 2x_1x_2-8x_1 = 2x_1(x_2-4)=0$

$f_{x_2} = x_1^2-2x_2$

① $x_1=0$

$x_2=0$

$|H|=16$

$f_{x_1x_1}=-8$

$\therefore f(0,0)=0$ = max.

② $x_2=4$

$x_1=\pm\sqrt{8}$

$|H|=-16+16-32<0$

鞍点

S.O.C.

$$|H| = \begin{vmatrix} 2x_2-8 & 2x_1 \\ 2x_1 & -2 \end{vmatrix} = -4x_2+16-4x_1^2$$

(3) $f(x,y) = 8x^2y$. s.t. $3x - y = 9$

$\mathcal{L} = 8x^2y + \lambda(9 - 3x + y)$

$\mathcal{L}_x = 16xy - 3\lambda = 0$ $\quad \frac{16xy}{8x^2} = \frac{-3}{1}$

$\mathcal{L}_y = 8x^2 + \lambda = 0$ $\quad 2xy = -3x^2$

$\mathcal{L}_\lambda = 9 - 3x + y = 0$ $\quad 3x^2 + 2xy = 0$

$x(3x + 2y) = 0$

① $x = 0$

$y = -9$

$(0, -9)$

$|\bar{H}| = 144 > 0$

Max $= f(0,-9) = 0$

② $x = -\frac{2}{3}y$

$y = -3$.

$x = 2$

$(2, -3)$

$|\bar{H}| = -192 + 48 = -144$

Min $= f(2,-3) = 8 \times 4 \times (-3)$

$= -96$

$$|\bar{H}| = \begin{vmatrix} 16y & 16x & -3 \\ 16x & 0 & 1 \\ -3 & 1 & 0 \end{vmatrix} = -48x - 48x - 16y$$

$$= -96x - 16y$$

2.

(1) $f(x.y.z) = x^2+6y^2+3z^2-2xy-4yz$

$$= (x.y.z)\begin{pmatrix}1 & -1 & 0\\ -1 & 6 & -2\\ 0 & -2 & 3\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}$$

$|H_1| = 1.\quad |H_2| = \begin{vmatrix}1 & -1\\ -1 & 6\end{vmatrix} = 6-1 = 5.$

$$|H_3| = \begin{vmatrix}1 & -1 & 0\\ -1 & 6 & -2\\ 0 & -2 & 3\end{vmatrix} = 18-3-4 = 11$$

$\therefore f(x.y.z)$ 为正定式.

(2) $f(x.y) = x^2+y^2 = (x.y)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}$

$|H_1| = 1 = |H_2|$

$f(x.y)$ 为半正定式.

(3) $f(x.y) = xy$

$$= (x.y.z)\begin{pmatrix}0 & \frac{1}{2} & 0\\ \frac{1}{2} & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}$$

$|H_1| = 0.\quad |H_2| = \begin{vmatrix}0 & \frac{1}{2}\\ \frac{1}{2} & 0\end{vmatrix} = -\frac{1}{4}\quad |H_3| = 0$

$f(x.y.z)$ 为不定式.

$f_x = y \qquad |H| = 0\cdot 0 - 1 = -1.$

$f_y = x$

$f_{xx} = 0 \quad f_{yy} = 0$

$f_{xy} = 1$