國立宜蘭大學 103 年度第一次微積分競試試題題解

第一部分:單選題 20 題，每題答對得 4 分，答錯或選項多於一個者倒扣一分，未作答者不予計分。

1. Evaluate $\lim_{x\to\infty}\frac{3^x+5^x}{3^{x+1}+5^{x+3}}=?$

(A)$\frac{1}{5}$ (B)$\frac{1}{25}$ (C)$\frac{1}{125}$ (D)$\frac{1}{625}$ (E)Does not exist.

Ans:

$$\lim_{x\to\infty}\frac{3^x+5^x}{3^{x+1}+5^{x+3}}=\lim_{x\to\infty}\frac{\left(\frac{3}{5}\right)^x+1}{3\left(\frac{3}{5}\right)^x+125}=\frac{1}{125}$$

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2. Evaluate $\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt{x}-1}=?$

(A)$\frac{2}{3}$ (B)1 (C)$\frac{3}{2}$ (D)0 (E)∞

Ans:

$$\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt{x}-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1}\cdot\frac{\sqrt[3]{x}+\sqrt[3]{x}+1}{\sqrt[3]{x}+\sqrt[3]{x}+1}=\lim_{x\to 1}\frac{(x-1)\cdot\left(\sqrt{x}+1\right)}{(x-1)\cdot\left(\sqrt[3]{x}+\sqrt[3]{x}+1\right)}=\frac{2}{3}\quad \#$$

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3. Evaluate $\lim_{x\to 1}\frac{1-\sqrt{x}}{x\sin\pi x}=?$

(A)$-\frac{2}{\pi}$ (B)0 (C)$\frac{2}{\pi}$ (D)$\frac{1}{2\pi}$ (E) does not exist

Ans:

$$\lim_{x\to 1}\frac{1-\sqrt{x}}{x\sin\pi x}=\lim_{x\to 1}\frac{-\frac{1}{2}x^{\frac{-1}{2}}}{\sin\pi x+\pi\cos\pi x}=\frac{-\frac{1}{2}}{\pi\times(-1)}=\frac{1}{2\pi}$$

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4. Evaluate $\frac{d}{dx}\left[\log_{\sqrt{5}}\sec^{-1}(1+x)\right]=?$

(A)$\frac{2}{\ln 5\cdot\sec^{-1}(1+x)\cdot\sqrt{1-(1+x)^2}}$ (B)$\frac{2}{\ln 5\cdot\sec^{-1}(1+x)+\sqrt{1-(1+x)^2}}$ (C)$\frac{5}{\ln 5\cdot\sec^{-1}(1+x)\cdot\sqrt{1-(1+x)^2}}$

(D) $\dfrac{2}{\ln 5\cdot \sec^{-1}(1-x)\cdot\sqrt{1-(1+x)^2}}$ (E) $\dfrac{\sqrt{5}}{\ln 5\cdot \sec^{-1}(1+x)\cdot\sqrt{1-(1+x)^2}}$

Ans:

$$\frac{d}{dx}\left[\log_{\sqrt{5}}\sec^{-1}(1+x)\right]=\frac{1}{\ln\sqrt{5}\sec^{-1}(1+x)}\cdot\frac{1}{\sqrt{1-(1+x)^2}}=\frac{1}{\frac{1}{2}\ln 5\cdot\sec^{-1}(1+x)\cdot\sqrt{1-(1+x)^2}}$$

$$=\frac{2}{\ln 5\cdot\sec^{-1}(1+x)\sqrt{1-(1+x)^2}}$$

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5. If $y=2^{\tan^{-1}x}+e^{x^{\sqrt{x}}}$, please find $\dfrac{dy}{dx}=?$

(A) $\dfrac{\ln 2\times 2^{\tan^{-1}x}}{1+x^2}+x^{\sqrt{x}}\left[\dfrac{1}{2\sqrt{x}}\ln x+\sqrt{x}\dfrac{1}{x}\right]$ (B) $\dfrac{\ln 2\times 2^{\tan^{-1}x}}{1-x^2}+x^{\sqrt{x}}\left[\dfrac{1}{2\sqrt{x}}\ln x+\sqrt{x}\dfrac{1}{x}\right]\cdot e^{x^{\sqrt{x}}}$

(C) $\dfrac{\ln 2\times 2^{\tan^{-1}x}}{1+x^2}+x^{\sqrt{x}}\left[\dfrac{1}{2\sqrt{x}}\ln x+\sqrt{x}\dfrac{1}{x}\right]\cdot e^{x^{\sqrt{x}}}$ (D) $\dfrac{\ln 2\times 2^{\tan^{-1}x}}{1-x^2}+x^{\sqrt{x}}\left[\dfrac{1}{\sqrt{x}}\ln x+\sqrt{x}\dfrac{1}{x}\right]\cdot e^{x^{\sqrt{x}}}$

(E) $\dfrac{\ln 2\times 2^{\tan^{-1}x}}{1-x}+x^{\sqrt{x}}\left[\dfrac{1}{2\sqrt{x}}\ln x+\sqrt{x}\dfrac{1}{x}\right]\cdot e^{x^{\sqrt{x}}}$

Ans:

$$Let: u=x^{\sqrt{x}}\Rightarrow \ln u=\sqrt{x}\ln x\Rightarrow\frac{1}{u}\cdot\frac{du}{dx}=\frac{1}{2\sqrt{x}}\ln x+\sqrt{x}\frac{1}{x}$$

$$\Rightarrow\frac{du}{dx}=x^{\sqrt{x}}\left[\frac{1}{2\sqrt{x}}\ln x+\sqrt{x}\frac{1}{x}\right]$$

$$Let: v=2^{\tan^{-1}x}$$

$$\Rightarrow\frac{dv}{dx}=\ln 2\times 2^{\tan^{-1}x}\cdot\left(\frac{1}{1+x^2}\right)$$

$$\Rightarrow\frac{dy}{dx}=\frac{dv}{dx}+\frac{du}{dx}=\frac{\ln 2\times 2^{\tan^{-1}x}}{1+x^2}+(\ln x)^{\sqrt{x}}\left[\frac{\ln(\ln x)}{2\sqrt{x}}+\frac{\sqrt{x}}{x\ln x}\right]$$

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6. Considering a function $\sin(xy)=4y\arccos(x-1)$, please find the equation of normal line at point $(1,2\pi)$.

(A) $y-2\pi=\dfrac{2\pi-1}{10\pi}(x-1)$ (B) $y-2\pi=\dfrac{10\pi}{1-2\pi}(x-1)$ (C) $y-2\pi=\dfrac{10\pi}{2\pi-1}(x-1)$ (D) $y-2\pi=\dfrac{1-2\pi}{10\pi}(x-1)$

(E) $y-2\pi=\dfrac{5\pi}{2\pi-1}(x-1)$

Ans:

$\sin(xy) = 4y\arccos(x-1)$

$\Rightarrow \frac{d}{dx}\sin(xy) = \frac{d}{dx}(4y\arccos(x-1))$

$\Rightarrow \cos(xy)\cdot\left(y + x\frac{dy}{dx}\right) = 4\frac{dy}{dx}\arccos(x-1) + 4y\frac{-1}{\sqrt{1+(x-1)^2}}$

$\Rightarrow \left.\frac{dy}{dx}\right|_{\substack{x=1\\y=2\pi}} = \frac{-10\pi}{1-2\pi}$...the slope of the tangent line

$\Rightarrow m = \frac{-1}{\frac{-10\pi}{1-2\pi}} = \frac{1-2\pi}{10\pi}$...the slope of the normal line

$\Rightarrow y - 2\pi = \frac{1-2\pi}{10\pi}(x-1)$

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7. If $f(x) = 1 + x + e^{-x}$, please evaluate $\left.\frac{d(f^{-1})}{dx}\right|_{x=2}$ =?

(A)0 (B)$\frac{1}{3}$ (C)$\frac{1}{2}$ (D)1 (E)Does not exist.

Ans:

$f(x) = 1 + x + e^{-x} = 2 \Rightarrow x = 0$

$\frac{d\left(f'(x)\right)^{-1}}{dx} = \frac{d}{dx(f'(x))} = \frac{1}{0+1-e^{x}} = \frac{1}{0}$ = Does not exist

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8. If $f(x)$ is continuous on [0 , 2] and differentiable in (0 , 2). Suppose that $f(0) = -3$ and $1 < f'(x) < 2$ for all x in (0 , 2). Find a possible value of $f(2)$.

(A)2 (B)3 (C)4 (D)5 (E) 6

Ans:

By The Mean value Theorem

$f'(x) = \frac{f(2) - f(0)}{2-0}$

$\Rightarrow 2 < f(2) - f(0) < 4$

$\Rightarrow 5 < f(2) < 7$

$\Rightarrow$ take $f(2) = 6$

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9. Evaluate $\frac{d}{dx}\int_{3x+x^2}^{0} t\sqrt{\left(1+t^2\right)^5}\,dt$ =?

(A) $-\left(3x+x^2\right)\sqrt{\left[1+\left(3x+x^2\right)^2\right]^5}\cdot\left(3x+x^2\right)$ (B) $-\left(3x+x^2\right)\sqrt{\left[1+\left(3x+x^2\right)^2\right]^5}\cdot\left(3+2x\right)$

(C) $-\left(3x+x^2\right)\sqrt{\left[1+\left(3x+x^2\right)^2\right]}\cdot\left(3+2x\right)$ (D) $\left(3x+x^2\right)\sqrt{\left[1+\left(3x+x^2\right)^2\right]^5}\cdot\left(3+2x\right)$

(E) $\left(3x+x^2\right)\sqrt{\left[1+\left(3x+x^2\right)^2\right]^5}\cdot\left(3x+x^2\right)$

Ans:

$$\frac{d}{dx}\int_{h(x)}^{g(x)} f(x)dx = f(g(x))\cdot g'(x) - f(h(x))\cdot h'(x)$$

$$\therefore \frac{d}{dx}\int_{3x+x^2}^{0} t\sqrt{\left(1+t^2\right)^5}\,dt = -\left(3x+x^2\right)\sqrt{\left[1+\left(3x+x^2\right)^2\right]^5}\cdot(3+2x) \quad \#$$

10.Evaluate $\int_{-10}^{10}\frac{\sin 6x}{\sqrt{1+x^{100}}}dx = ?$

(A)-2 (B)-1 (C)0 (D)1 (E) 2

Ans:

$$Let: f(x) = \frac{\sin 6x}{\sqrt{1+x^{100}}}$$

$$\because f(-x) = \frac{\sin(-6x)}{\sqrt{1+(-x)^{100}}} = -\frac{\sin 6x}{\sqrt{1+x^{100}}} = -f(x)$$

$f(x)$ is odd function

$$\therefore \int_{-10}^{10}\frac{\sin 6x}{\sqrt{1+x^{100}}}dx = 0$$

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11. If $f(x)$ is a continuous function defined in R, and $\int_0^{x^2} f(t)dt = x\,arc\sec \pi x$, please find $f(9)$.

(A) $\frac{3\pi}{\sqrt{1-(3\pi)^2}}-1$ (B) $\frac{\pi}{2\sqrt{1-(3\pi)^2}}-\frac{1}{6}$ (C) $\frac{\pi}{2\sqrt{1-(3\pi)^2}}+\frac{1}{6}$ (D) $\frac{3\pi}{\sqrt{1-(3\pi)^2}}+1$

(E) $\frac{\pi}{\sqrt{1-(3\pi)^2}}-\frac{1}{6}$

Ans:

$$\int_0^{x^2} f(t)dt = x\,arc\sec \pi x$$

$$\Rightarrow \frac{d}{dx}\int_0^{x^2} f(t)dt = \frac{d}{dx}\left(x\,arc\sec \pi x\right)$$

$$\Rightarrow f\left(x^2\right)\times 2x = arc\sec \pi x + \frac{\pi x}{\sqrt{1-(\pi x)^2}}$$

$$\Rightarrow x = 3 \Rightarrow f(9)\times 6 = arc\sec 3\pi + \frac{3\pi}{\sqrt{1-(3\pi)^2}} = \frac{3\pi}{\sqrt{1-(3\pi)^2}} - 1$$

$$f(9) = \frac{\pi}{2\sqrt{1-(3\pi)^2}} - \frac{1}{6}$$

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12. Evaluate $\int \arccos\sqrt{x}dx = ?$

(A) $\cos^{-1}\sqrt{x}+2\sqrt{1-x}-\frac{2}{3}(1-x)^{\frac{3}{2}}$ (B) $x\cos^{-1}\sqrt{x}-2\sqrt{1-x}-\frac{2}{3}(1-x)^{\frac{3}{2}}$

(C) $x\cos^{-1}\sqrt{x}+2\sqrt{1-x}-\frac{2}{3}(1-x)^{\frac{3}{2}}$ (D) $x\cos^{-1}\sqrt{x}+\sqrt{1-x}-\frac{2}{3}(1-x)^{\frac{3}{2}}$

(E) $x\cos^{-1}\sqrt{x}+2\sqrt{1-x}+\frac{2}{3}(1-x)^{\frac{3}{2}}$

Ans:

$$\int \cos^{-1}\sqrt{x}dx = x\cos^{-1}\sqrt{x} - \int x\left(\frac{-1}{\sqrt{1-x}}\right)dx$$

$$= x\cos^{-1}\sqrt{x} + \int \frac{x}{\sqrt{1-x}}dx$$

$$= x\cos^{-1}\sqrt{x} + \int \frac{x}{\sqrt{1-x}}d(1-x)$$

$$Let: u=(1-x) \Rightarrow x = 1-u \Rightarrow \int \frac{x}{\sqrt{1-x}}d(1-x) = \int \frac{1-u}{\sqrt{u}}d(u) = 2\sqrt{u}-\frac{2}{3}u^{\frac{3}{2}} = 2\sqrt{1-x}-\frac{2}{3}(1-x)^{\frac{3}{2}}$$

$$x\cos^{-1}\sqrt{x} + \int \frac{x}{\sqrt{1-x}}d(1-x) = x\cos^{-1}\sqrt{x} + 2\sqrt{1-x} - \frac{2}{3}(1-x)^{\frac{3}{2}}$$

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13. Evaluate $\int \tan^4 xdx = ?$

(A) $(\sec^2 x-1)^2+c$ (B) $\frac{1}{3}\tan^3 x+\tan x+x+c$ (C) $\frac{1}{3}\tan^3 x-\tan x-x+c$

(D) $(\sec^2 x+1)^2+c$ (E) $\frac{1}{3}\tan^3 x-\tan x+x+c$

Ans:

$$\int \tan^4 xdx = \int \left(\sec^2 x-1\right)^2 dx = \int \left(\sec^4 x - 2\sec^2 x + 1\right)dx$$

$$= \int \sec^2 xd\left(\tan x\right) - 2\int \sec^2 xdx + \int 1dx$$

$$= \int \left(1+\tan^2 x\right)d\left(\tan x\right) - 2\tan x + x + c$$

$$= \tan x + \frac{1}{3}\tan^3 x - 2\tan x + x + c$$

$$= \frac{1}{3}\tan^3 x - \tan x + x + c \quad \#$$

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14. Evaluate $\int \frac{1}{1+\sin x}dx = ?$

(A) $\tan x + C$ (B) $\ln|1+\sin x|+C$ (C) $\tan x+\sec x+C$ (D) $\tan x-\sec x+C$

(E) $\sec x + C$

Ans:

$$\int \frac{1}{1+\sin x}dx = \int \frac{1-\sin x}{(1-\sin x)(1+\sin x)}dx = \int \frac{1-\sin x}{\cos^2 x}dx = \int (\sec^2 x - \sec x\tan x)dx = \tan x - \sec x + C$$

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15. Evaluate $\int \frac{1}{3+\cos x-\sin x}dx = ?$(hint: let $tan\frac{x}{2} = $ t)

(A) $\frac{\sqrt{7}}{2}\tan^{-1}\left[\frac{\sqrt{7}}{2}\left(\tan\frac{x}{2}-\frac{1}{2}\right)\right]+C$ (B) $\frac{\sqrt{7}}{2}\tan^{-1}\left[\frac{2}{\sqrt{7}}\left(\tan\frac{x}{2}-\frac{1}{2}\right)\right]+C$ (C) $\frac{2}{\sqrt{7}}\tan^{-1}\left[\frac{2}{\sqrt{7}}\left(\tan\frac{x}{2}-\frac{1}{2}\right)\right]+C$

(D) $\frac{2}{\sqrt{7}}\tan^{-1}\left[\frac{\sqrt{7}}{2}\left(\tan\frac{x}{2}-\frac{1}{2}\right)\right]+C$ (E) $\frac{4}{\sqrt{7}}\tan^{-1}\left[\frac{\sqrt{7}}{2}\left(\tan\frac{x}{2}-\frac{1}{2}\right)\right]+C$

Ans:

$$\tan\frac{x}{2}=t\Rightarrow \sin x=\frac{2t}{1+t^2},\cos x=\frac{1-t^2}{1+t^2},dx=\frac{2}{1+t^2}dt$$

$$\int\frac{1}{3+\cos x-\sin x}dx=\int\frac{1}{3+\frac{1-t^2}{1+t^2}-\frac{2t}{1+t^2}}\times\frac{2}{1+t^2}dt=\int\frac{1}{t^2-t+2}dt=\int\frac{1}{\left(\frac{\sqrt{7}}{2}\right)^2+\left(t-\frac{1}{2}\right)^2}dt$$

$$=\frac{2}{\sqrt{7}}\int\frac{1}{1+\left[\frac{2}{\sqrt{7}}\left(t-\frac{1}{2}\right)\right]^2}d\left[\frac{2}{\sqrt{7}}\left(t-\frac{1}{2}\right)\right]=\frac{2}{\sqrt{7}}\tan^{-1}\left[\frac{2}{\sqrt{7}}\left(t-\frac{1}{2}\right)\right]+C=\frac{2}{\sqrt{7}}\tan^{-1}\left[\frac{2}{\sqrt{7}}\left(\tan\frac{x}{2}-\frac{1}{2}\right)\right]+C$$

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16. Find the volume of the solid of the revolution formed by revolving the region bounded by $y=1-\frac{x^2}{16}$ and the x-axis $0\le x\le 4$ about the x-axis.

(A) $\frac{32}{15}\pi$ (B) 2π (C) $\frac{28}{15}\pi$

(D) $\frac{26\pi}{15}$ (E) $\frac{8}{5}\pi$

Ans:

$$V=\pi\int_a^b r(x)^2dx=\pi\int_0^4(1-\frac{x^2}{16})^2dx=\pi\int_0^4(1-\frac{x^2}{8}+\frac{x^4}{256})dx$$

$$=\pi[x-\frac{x^3}{24}+\frac{x^5}{1280}]_0^4=\frac{32}{15}\pi$$

17. Evaluate $\int\frac{2x^3-4x^2-15x+5}{x^2-2x-8}dx=?$

(A) $x^2+\frac{3}{2}\ln|x-4|-\ln|x+2|+c$ (B) $x^2+\frac{3}{2}\ln|x-2|-\frac{1}{2}\ln|x+4|+c$

(C) $x^2+\ln|x-4|-\frac{1}{2}\ln|x+2|+c$ (D) $x^2+\frac{3}{2}\ln|x-4|+\ln|x+2|+c$

(E) $x^2+\frac{3}{2}\ln|x-4|-\frac{1}{2}\ln|x+2|+c$

Ans:

$$\frac{2x^3-4x^2-15x+5}{x^2-2x-8}=2x+\frac{x+5}{(x-4)(x+2)}=2x+\frac{A}{x-4}+\frac{B}{x+2}$$

$$x+5=A(x-2)+B(x-4)\to A=\frac{3}{2}\quad B=-\frac{1}{2}$$

$$\int\frac{2x^3-4x^2-15x+5}{x^2-2x-8}\,dx=\int\left(2x+\frac{3/2}{x-4}-\frac{1/2}{x+2}\right)dx$$

$$=x^2+\frac{3}{2}\ln|x-4|-\frac{1}{2}\ln|x+2|+c$$

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18. Evaluate $\int\frac{1}{a^4-x^4}dx=?$

(A) $\frac{1}{2a^3}\left[\ln|a+x|+\ln|a-x|+2\tan^{-1}\frac{x}{a}\right]+c$ (B) $\frac{1}{4a^3}\left[\ln|a+x|-\ln|a-x|+2\tan^{-1}\frac{x}{a}\right]+c$

(C) $\frac{3}{4a^3}\left[\ln|a+x|-\ln|a-x|+\frac{1}{a}\tan^{-1}\frac{x}{a}\right]+c$ (D) $\frac{x}{4a^3}\left[\ln|a+x|-\ln|a-x|+\frac{x}{a}\tan^{-1}\frac{x}{a}\right]+c$

(E) $\frac{1}{4a^3}\left[\ln|2a+x|+\ln|a-x|+\tan^{-1}\frac{x}{a}\right]+c$

$$\int\frac{1}{a^4-x^4}dx=\int\left[\frac{1}{4a^3}\left(\frac{1}{a-x}+\frac{1}{a+x}\right)+\frac{1}{2a^2}\cdot\frac{1}{a^2+x^2}\right]dx$$

$$=\frac{1}{4a^3}\int\left[\frac{1}{a-x}+\frac{1}{a+x}+\frac{2}{a^2+x^2}\right]dx=\frac{1}{4a^3}\left[\ln|a+x|-\ln|a-x|+2\tan^{-1}\frac{x}{a}\right]+C\quad (C \text{ is constant})$$

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19. Evaluate $\int_1^7\frac{\ln x}{x^3}dx=?$

(A) $\frac{1872}{625}$ (B) $\frac{-1}{3}\left[\frac{2\ln 5+1}{50}-\frac{1}{2}\right]$ (C) $\frac{24-2\ln 7}{100}$ (D) $\frac{-1}{2}\left[\frac{2\ln 7+1}{50}-\frac{1}{4}\right]$ (E) $\frac{1}{2}\left[\frac{2\ln 7+1}{50}-\frac{1}{2}\right]$

Ans:

$Let: u=\ln x \Leftrightarrow x=e^u$

$du=\frac{1}{x}dx$

$$\int_1^7\frac{\ln x}{x^3}dx=\int_1^7\frac{1}{x}\cdot\frac{\ln x}{x^2}dx=\int_0^{\ln 7}\frac{u}{e^{2u}}du=\int_0^{\ln 7}u\cdot e^{-2u}du=\frac{-1}{2}\int_0^{\ln 7}u\cdot de^{-2u}=\frac{-1}{2}\left[u\cdot e^{-2u}-\int e^{-2u}du\right]_0^{\ln 7}$$

$$=\frac{-1}{2}\left[u\cdot e^{-2u}+\frac{e^{-2u}}{2}\right]_0^{\ln 7}=\frac{-1}{2}\left[\frac{2u+1}{2e^{2u}}\right]_0^{\ln 7}=\frac{-1}{2}\left[\frac{2\ln x+1}{2x^2}\right]_1^7=\frac{-1}{2}\left[\frac{2\ln 7+1}{50}-\frac{1}{2}\right]=\frac{24-2\ln 7}{100}$$

20.Evaluate $\lim_{x\to 4}\frac{x\int_4^x\frac{\cos t}{t}dt}{x-4}=?$

(A) $\sin 4$ (B) $\sin 8$ (C) $\cos 8$ (D) $\cos 4$ (E) $\tan 8$

Ans:

$$\lim_{x\to 4}\frac{x\int_4^x\frac{\cos t}{t}dt}{x-4}=\lim_{x\to 4}\frac{1\cdot\int_4^x\frac{\cos t}{t}dt+x\cdot\frac{\cos x}{x}}{1}=0+\cos 4=\cos 4$$

第二部分:非選題共一題，計 20 分，請將答案用藍黑色原子筆寫在答案紙上。

1. Assume that $|\varepsilon| << 1$, determine a, b, a', b' for the solutions of this function $x^2 + (\varepsilon - 4)x + (3 + 2\varepsilon) = 0$;

$x^{(1)} \cong 1 + a\varepsilon + b\varepsilon^2 + O(\varepsilon^3)$

$x^{(2)} \cong 3 + a'\varepsilon + b'\varepsilon^2 + O'(\varepsilon^3)$

$x^{(1)}, x^{(2)}$ are the solutions of the function, where the remainder terms $O(\varepsilon^3), O'(\varepsilon^3)$ are postulated to be significantly less than prior terms.

ANS：

$x = 1 + a\varepsilon + b\varepsilon^2 + \cdots$

$x^2 = 1 + 2a\varepsilon + (a^2 + 2b)\varepsilon^2 + \cdots$

$(\varepsilon - 4)x = (-4) + (1 - 4a)\varepsilon + (a - 4b)\varepsilon^2 + \cdots$

$\therefore x^2 + (\varepsilon - 4)x + (3 + 2\varepsilon) = (-4) + (1 - 4a)\varepsilon + (a - 4b)\varepsilon^2 + 1 + 2a\varepsilon + (a^2 + 2b)\varepsilon^2 + (3 + 2\varepsilon) + \cdots = 0$

$= (3 - 2a)\varepsilon + (a^2 + a - 2b)\varepsilon^2 + \cdots = 0$

$3 - 2a = 0, a = \dfrac{3}{2}$

$a^2 + a - 2b = 0, b = \dfrac{15}{8}$

$x = 3 + a'\varepsilon + b'\varepsilon^2 + \cdots$

$x^2 = 9 + 6a'\varepsilon + (a'^2 + 2b')\varepsilon^2 + \cdots$

$(\varepsilon - 4)x = (-12) + (3 - 4a')\varepsilon + (a' - 4b')\varepsilon^2 + \cdots$

$\therefore x^2 + (\varepsilon - 4)x + (3 + 2\varepsilon) = 9 + 6a'\varepsilon + (a'^2 + 2b')\varepsilon^2 + (-12) + (3 - 4a')\varepsilon + (a' - 4b')\varepsilon^2 + (3 + 2\varepsilon) + \cdots = 0$

$= (2a' + 5)\varepsilon + (a'^2 + a' + 2b)\varepsilon^2 + \cdots = 0$

$2a' + 5 = 0, a' = \dfrac{-5}{2}$

$a'^2 + a' - 2b' = 0, b' = \dfrac{-15}{8}$