

微 積 分 期 中 考

1. Find the **absolute** extrema of the function on the closed interval. (10%)

(a) $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3, [-1, 2]$ (b) $y = \tan\left(\frac{\pi x}{8}\right), [0, 2]$

2. Find the open interval(s) on which the function is increasing or decreasing, and to identify all **relative** extrema. (20%)

(a) $f(x) = x^2 - 4x$ (b) $f(x) = (x - 3)^{1/3}$
(c) $f(x) = \frac{x^2 - 2x + 1}{x + 1}$ (d) $f(x) = \frac{\sin x}{1 + \cos^2 x}, (0, 2\pi)$

3. Find the points of inflection and discuss the concavity of the graph of the function. (10%)

(a) $f(x) = (x - 2)^3(x - 1)$ (b) $f(x) = 2\sin x + \sin 2x, [0, 2\pi]$

4. Find all relative extrema. Use the Second Derivative Test where applicable. (10%)

(a) $f(x) = \sqrt{x^2 + 1}$ (b) $f(x) = 2\sin x + \cos 2x, [0, \pi]$

5. Analyze and sketch a graph of the function. Label any intercepts, relative extrema, points of inflection, and asymptotes. (20%)

$$y = \frac{1}{x - 2} - 3$$

Find the derivative $\frac{dy}{dx}$ or f' . (30%)

6. $y = 2x^3 + 6x^2 - 1$

7. $y = \frac{1 - \sin x}{\cos x}$

8. $f(t) = \left(\frac{t^2}{t^3 + 2}\right)^2$

9. $y = \sin(\tan 2x)$

10. $x^3 - 3x^2y + 2xy^2 = 12$

11. $\cot y = x - y$