105 学年度第1学期微積分一期中考解答

105.11.14

1. (12%)

$f(x) = x^2 + x - 5$

(1) $f(2) = 4 + 2 - 5 = 1$

(2) $f(2+h) = (2+h)^2 + (2+h) - 5 = h^2 + 5h + 1$

(3) $\dfrac{f(2+h)-f(2)}{h} = \dfrac{h^2+5h+1-1}{h} = h + 5$

(4) $\lim\limits_{h\to 0} \dfrac{f(2+h)-f(2)}{h} = \lim\limits_{h\to 0}(h+5) = 5.$

2. (16%)

(1) $\lim\limits_{n\to\infty} \dfrac{n!}{n^n} = \lim\limits_{n\to\infty}\left(\dfrac{1}{n}\times\dfrac{2}{n}\times\dfrac{3}{n}\times\cdots\times\dfrac{n}{n}\right) = 0.$

(2) $\lim\limits_{x\to\infty}\left(\sqrt{x^2+1} - x\right) = \lim\limits_{x\to\infty}\left(\dfrac{(\sqrt{x^2+1}+x)(\sqrt{x^2+1}-x)}{\sqrt{x^2+1}+x}\right)$

$= \lim\limits_{x\to\infty}\left(\dfrac{x^2+1-x^2}{\sqrt{x^2+1}+x}\right) = 0.$

(3) $\lim\limits_{h\to 0}\dfrac{\sqrt{h+4}-2}{h} = \lim\limits_{h\to 0}\dfrac{(\sqrt{h+4}+2)(\sqrt{h+4}-2)}{h(\sqrt{h+4}+2)} = \lim\limits_{h\to 0}\dfrac{h+4-4}{h(\sqrt{h+4}+2)}$

$= \lim\limits_{h\to 0}\dfrac{1}{\sqrt{h+4}+2} = \dfrac{1}{4}.$

(4) $\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2-3x+1}}{2x+11} = \lim\limits_{x\to-\infty}\dfrac{-1}{2+\frac{11}{x}} = -\dfrac{1}{2}.$

(5) $\lim_{x\to 1} \frac{|x-1|}{x-1} = \begin{cases} \lim_{x\to 1^+} \frac{x-1}{x-1} = \lim_{x\to 1^+} (1) = 1 \\ \lim_{x\to 1^-} \frac{-(x-1)}{x-1} = \lim_{x\to 1^-} (-1) = -1 \end{cases}$

$\therefore \lim_{x\to 1} \frac{|x-1|}{x-1}$ 不存在.

(6) $\lim_{x\to 2} \frac{2x^3-7x^2+16x+2}{x^3+3x^2-16x-5} = \lim_{x\to 2} \frac{2-\frac{7}{x}+\frac{16}{x^2}+\frac{2}{x^3}}{1+\frac{3}{x}-\frac{16}{x^2}-\frac{5}{x^3}} = \frac{22}{17}$

(7) $\lim_{x\to 1} \frac{x^2-3x+2}{x^2-5x+4} = \lim_{x\to 1} \frac{(x-1)(x-2)}{(x-1)(x-4)} = \lim_{x\to 1} \frac{x-2}{x-4} = \frac{1}{3}$

(8) $\lim_{x\to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3}\right) = \lim_{x\to 1} \left(\frac{1+x+x^2-3}{1-x^3}\right) = \lim_{x\to 1} \left(\frac{x^2+x-2}{1-x^3}\right)$

$= \lim_{x\to 1} \frac{(x+2)(x-1)}{-(x-1)(x^2+x-1)} = \lim_{x\to 1} \left(\frac{-(x+2)}{x^2+x-1}\right)$

$= \frac{-3}{1} = -3.$

3. $f(x) = \begin{cases} 3x^2+a & x \le 0 \\ 4x-b & 0 \le x \le 1 \\ \sqrt{x+8} & x > 1. \end{cases}$

① $0+a = 0-b$ ② $4-b = \sqrt{9} = 3$

$a = -b$ $b = 1.$ $\therefore a = -1.$ $b = 1.$

4. 証明 (略)

5. $f(x) = \frac{36}{(3-x)^2} = 36(3-x)^{-2}$

$f'(x) = +72(3-x)^{-3}$ $m = f'(0) = 72(3)^{-3} = \frac{72}{27} = \frac{8}{3}$

$y-4 = \frac{8}{3}(x-0)$ $3y-12 = 8x$ $\therefore 8x+3y = 12$ p2

6. $a \neq 0$

$y = ax^2 + bx + c$. $y = bx^2 + ax + c$

$m_1 = 2ax + b$. $m_2 = 2bx + a$

$m_1 = m_2$. $2ax + b = 2bx + a$

$$x = \frac{a-b}{2a-2b} = \frac{1}{2}.$$

7. (1) $y = \sqrt{x-8}$. $D: \{x \geq 8.\ x \in R\}$ $R = \{y \geq 0.\ y \in R\}$.

(2) $y = \sqrt[3]{x-8}$ $D: \{x \in R\}$ $R: \{y \in R\}$.

(3) $y = \frac{1}{|x-1|}$ $D: \{x \neq 1.\ x \in R\}$ ~~$R: \{y \neq 0.\ y \in R\}$~~.

$R: \{y > 0, y \in R\}$

8. $f(x) = x^4 - 2x^2$.

$f'(x) = 4x^3 - 4x$

$\therefore f'(-1) = -4 + 4 = 0.$

$f'(0) = 0.$

9. $f(x) = x^3$. $g(x) = x^2 + x + 1$.

$$\frac{d}{dx} f(g(x)) = \frac{d}{dx}\left((x^2+x+1)^3\right) = 3(x^2+x+1)^2(2x+1)$$

$$= 3(2x+1)(x^2+x+1)^2.$$

10.

(1) $f(x) = \frac{1}{(\sqrt{x^2+11})^7} = (\sqrt{x^2+11})^{-7} = (x^2+11)^{-\frac{7}{2}}$

$f'(x) = -\frac{7}{2}(x^2+11)^{-\frac{9}{2}}(2x) = -7x(x^2+11)^{-\frac{9}{2}}$.

(2) $f(x) = x\sqrt{2x+3}$

$f'(x) = \sqrt{2x+3} + (x)(\frac{1}{2})(2x+3)^{-\frac{1}{2}}(2)$

$= (2x+3)^{\frac{1}{2}} + x(2x+3)^{-\frac{1}{2}}$

(3) $f(x) = \sqrt{x} - \frac{1}{\sqrt{x}} = x^{\frac{1}{2}} - x^{-\frac{1}{2}}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}} = \frac{1}{2}(x^{-\frac{1}{2}} + x^{-\frac{3}{2}})$

(4) $f(x) = (3x^2+1)^2$

$f'(x) = 2(3x^2+1)(6x) = 12x(3x^2+1)$

11. $w = ay + 2$. $y = bx^2 + cx$

$\frac{dw}{dx} = \frac{dw}{dy} \cdot \frac{dy}{dx} = (a)(2bx+c)$

12. (1) $f(x) = 2x^2+3x+1$

$f'(x) = 4x+3$. $f''(x) = 4$

(2) $f(x) = \frac{1}{1-x} = -(x-1)^{-1}$

$f'(x) = (x-1)^{-2}$ $f''(x) = -2(x-1)^{-3}$.