國立宜蘭大學 105 學年度第 2 學期期末考試試題紙			共 1 頁　106.06.22
考 試 科 目	班級	學號	姓　名
微積分二			

1.(24%)求以下各三角函數的導數

(1) $y=2\sin^2 x-\cos 2x$　(2) $y=\ln(\sin 2x)$　(3) $y=e^{x^2}\sec x$

(4) $y=e^{x^2}\sec x$　(5) $f(x)=\dfrac{x\cos x}{\sin x}$　(6) $y=(\sec \pi x)^3$

2.(15%)(1) $\int \sin^2 x\cos^3 x dx$　(2) $\dfrac{\sec x\tan x}{\sec x-1}dx$　(3) $\int (2\sin x+3\cos x)dx$

3.(10%)求 $\dfrac{dy}{dx}$：(1) $xy=\tan y$　(2) $x\cos y+y\sin x=5$

4.(10%)求 $y=2\cos x$ 在點 $(\frac{\pi}{3},1)$ 的切線方程式.

5.(5%)若已知 $u=\dfrac{x}{y}+\dfrac{y}{z}+\dfrac{z}{x}$，求 $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}+z\dfrac{\partial u}{\partial z}$.

6.(10%) $f(x,y,z)=x^2+3y^2-3xy+4yz+6z^2$, please
(1)justify this function has maximum or minimum. (2)calculate its extreme value.

7.(10%)if $f(x,y)=xy$,which is subject to $x+y=100$,please use the Largrange multiplier method to find the maximum or minimum point and verify the extreme value.

8.(10%)某公司製造兩種球鞋：跑步鞋和籃球鞋。假設賣出 x_1 單位的跑步鞋和 x_2 單位的籃球鞋的總收入是 $TR=-5x_1^2-8x_2^2-2x_1x_2+42x_1+102x_2$，其中 x_1 和 x_2 的單位各為千雙，求使收入為最大的 x_1 和 x_2.

9.(10%)請判斷以下數列為發散或是收斂？如果為收斂，請找出收斂值.(1) $a_n=\dfrac{5}{n}$　(2) $a_n=\dfrac{2^n}{n!}$.

105学年度第二学期微積分二期末考解答

106.6.19

1. 24% 求導數

(1) $y = 2\sin^2 x - \cos 2x.$ $\frac{dy}{dx} = 4\sin x \cos x + 2\sin 2$
$= 4\sin 2x$

(2) $y = \ln(\sin 2x)$ $\frac{dy}{dx} = \frac{2\cos 2x}{\sin 2x} = 2\cot 2x$

(3) $y = e^{x^2}\sec x$ $\frac{dy}{dx} = e^{x^2}(2x)\cdot\sec x + e^{x^2}\cdot\tan x \sec$
$= e^{x^2}\sec x(2x + \tan x)$

(4) $y = \sin^2(x^2 - 2)$ $\frac{dy}{dx} = 2\sin(x^2-2)\cos(x^2-2)\cdot 2x$
$= 4x\sin(x^2-2)\cos(x^2-2)$
$= 2x\sin(2x^2-4)$

(5) $f(x) = \frac{x\cos x}{\sin x} = x\cdot\cot x$ $\frac{dy}{dx} = \cot x - x\csc^2 x$

(6) $y = (\sec \pi x)^3$ $\frac{dy}{dx} = 3\pi(\sec\pi x)^2\cdot\sec\pi x\tan\pi x$
$= 3\pi(\sec\pi x)^3\tan\pi x$

2.

5% (1) $\int \sin^2 x \cos^3 x \, dx$

$u = \sin x$

$du = \cos x$

$= \int \sin^2 x \, (1 - \sin^2 x) \cos x \, dx$

$= \int (\sin^2 x - \sin^4 x) \cos x \, dx$

$= \int (u^2 - u^4) \, du$

$= \dfrac{\sin^3 x}{3} - \dfrac{\sin^5 x}{5} + C$

5% (2) $\int \dfrac{\sec x \tan x}{\sec x - 1} \, dx$

$u = \sec x - 1$

$du = \sec x \tan x \, dx$

$= \int \dfrac{1}{u} \, du$

$= \ln |\sec x - 1| + C$

5% (3) $\int (2 \sin x + 3 \cos x) \, dx$

$= -2 \cos x + 3 \sin x + C.$

3. 求 $\frac{dy}{dx}$

5% (1) $xy = \tan y$

$$x\frac{dy}{dx} + y = (\sec^2 y)\frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{-y}{x - \sec^2 y}$$

5% (2) $x\cos y + y\sin x = 5$

$$\cos y - x\sin y \cdot \frac{dy}{dx} + \sin x\frac{dy}{dx} + y\cos x = 0$$

$$(\sin x - x\sin y)\frac{dy}{dx} = -\cos y - y\cos x$$

$$\frac{dy}{dx} = \frac{\cos y + y\cos x}{x\sin y - \sin x}$$

4. 10% $y = 2\cos x$

$$\frac{dy}{dx} = m = -2\sin x$$

$$\left.\frac{dy}{dx}\right|_{(\frac{\pi}{3}, 1)} = -2\sin\frac{\pi}{3} = -2\cdot\frac{\sqrt{3}}{2} = -\sqrt{3}.$$

切線方程式 : $y - 1 = -\sqrt{3}\left(x - \frac{\pi}{3}\right)$

5. $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$

5%

$x \cdot \frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$

$= x\left(\frac{1}{y} - \frac{z}{x^2}\right) + y\left(-\frac{x}{y^2} + \frac{1}{z}\right) + z\left(-\frac{y}{z^2} + \frac{1}{x}\right)$

$= 0$

6. $f(x, y, z) = x^2 + 3y^2 - 3xy + 4yz + 6z^2$

10%

$$\left.\begin{aligned} f_x &= 2x - 3y = 0 \\ f_y &= 6y - 3x + 4z = 0. \\ f_z &= 4y + 12z = 0 \end{aligned}\right\} \quad \begin{aligned} x^* &= 0 \\ y^* &= 0 \\ z^* &= 0 \end{aligned}$$

4%

$f(x, y, z) = (x, y, z)\begin{pmatrix} 1 & -\frac{3}{2} & 0 \\ -\frac{3}{2} & 3 & 2 \\ 0 & 2 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$|H_1| = 1. \quad |H_2| = 3 - \frac{9}{4} = \frac{3}{4} > 0$

$|H_3| = 18 - 4 - \frac{54}{4} = \frac{1}{2} > 0$

$\therefore f(0, 0, 0) = 0$: minimum.

3% 3%

7. $f(x, y) = xy$

s.t. $x + y = 100$

〈另解〉

$y = 100 - x$

$f(x, y) = x(100 - x)$

$= -x^2 + 100x$

$= -(x-50)^2 + 2500$

$f(x, y) \leq 2500.$

$\mathcal{L} = xy + \lambda(100 - x - y)$

10%

$$\left.\begin{aligned} \mathcal{L}_x &= y - \lambda = 0 \\ \mathcal{L}_y &= x - \lambda = 0 \\ \mathcal{L}_\lambda &= 100 - x - y = 0 \end{aligned}\right\} \quad \lambda^* = 50, \quad x^* = y^* = 50$$

$|H_2| = -1 < 0$

$|\bar{H}| = 2 > 0$

$$|\bar{H}| = \begin{vmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ -1 & -1 & 0 \end{vmatrix} = 0 + 1 + 1 = 2$$

$f(50, 50) = 2500 = \text{maximum}$

8. $TR = -5x_1^2 - 8x_2^2 - 2x_1x_2 + 42x_1 + 102x_2$

10%

$$\left.\begin{aligned} TR_1 &= -10x_1 - 2x_2 + 42 = 0 \\ TR_2 &= -16x_2 - 2x_1 + 102 = 0 \end{aligned}\right\} \quad x_1^* = 3, \quad x_2^* = 6$$

$$|H| = \begin{vmatrix} -10 & -2 \\ -2 & -16 \end{vmatrix} = 160 - 4 = 156$$

∴ TR為最大下的 $(x_1^*\ x_2^*) = (3, 6)$

9.

5% (1) $a_n = \frac{5}{n}$

$\lim_{n\to\infty} a_n = 0 \quad \therefore$ 為收斂

5% (2) $a_n = \frac{2^n}{n!}$

$= \frac{2\times2\times\cdots\times2}{1\times2\times3\times\cdots\times n}$

$= \frac{2}{1}\times\frac{2}{2}\times\frac{2}{3}\times\frac{2}{4}\times\cdots\times\frac{2}{n}$

$\lim_{n\to\infty} a_n = 0$

$\therefore$ 為收斂