國立宜蘭大學 105 學年度第 2 學期 期末 考試試題紙			第 頁
考試科目	班級	學號	姓名
微積分二		Solution	

Multiple choices (50 points)

1. (b) Find the radius of convergence of the power series $\sum_{n=0}^{\infty} 3(x-2)^n$. (a) ∞ (b) 1 (c) 2 (d) 3

2. (d) Consider the function given by $f(x)=\sum_{n=1}^{\infty}\frac{x^n}{n}$. Find the intervals of convergence for $\int f(x)dx$. (a) $(-1,1)$ (b) $(-1,1]$ (c) $[-1,1)$ (d) $[-1,1]$

3. (b) Find the interval of convergence of the power series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-c)^n}{nc^n}$. (a) $(0,2c)$ (b) $(0,2c]$ (c) $[0,2c)$ (d) $[0,2c]$

4. (c) Find a power series for $f(x)=\frac{4}{x+2}$, centered at 0. (a) $\sum_{n=0}^{\infty}4(-\frac{x}{2})^n$ (b) $\sum_{n=0}^{\infty}(-\frac{x}{2})^n$ (c) $\sum_{n=0}^{\infty}2(-\frac{x}{2})^n$ (d) $\sum_{n=0}^{\infty}(-x)^n$

5. (b) Find a power series for $f(x)=\frac{4x-7}{2x^2+3x-2}$, centered at 0. (a) $\sum_{n=0}^{\infty}\frac{3(-1)^n}{2^{n+1}}x^n$ (b) $\sum_{n=0}^{\infty}[\frac{3(-1)^n}{2^{n+1}}+2^{n+1}]x^n$ (c) $\sum_{n=0}^{\infty}2^{n+1}x^n$ (d) $\sum_{n=0}^{\infty}[3(-1)^n+2^{n+1}]x^n$

Calculation (50 points)

1. (a) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ when $xz+yz+xy=0$. (7 points)

   (b) Find $\frac{d^2w}{dt^2}$ when $t=0$ for $w=\arctan(2xy)$ where $x=\cos t$, $y=\sin t$. (7 points)

(a) $F_x = z+y$, $F_y = z+x$, $F_z = x+y$

$$\frac{\partial z}{\partial x} = -\frac{z+y}{x+y}, \quad \frac{\partial z}{\partial y} = -\frac{z+x}{x+y}$$

(b) $$\frac{dw}{dt} = \frac{\partial w}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial w}{\partial y}\cdot\frac{dy}{dt} = \frac{2y}{1+(2xy)^2}\cdot(-\sin t) + \frac{2x}{1+(2xy)^2}\cdot\cos t$$

$$= \frac{-2\sin^2 t}{1+4\cos^2 t\sin^2 t} + \frac{2\cos^2 t}{1+4\cos^2 t\sin t} = \frac{2\cos^2 t - 2\sin^2 t}{1+4\cos^2 t\sin^2 t}$$

$$\frac{d^2w}{dt^2} = \frac{(1+4\cos^2 t\sin^2 t)(-8\cos t\sin t)-(2\cos^2 t-2\sin^2 t)(8\cos^3 t\sin t-8\sin^3 t\cos t)}{(1+4\cos^2 t\sin^2 t)^2}$$

$$= \frac{-8\cos t\sin t(1+2\sin^4 t+2\cos^4 t)}{(1+4\cos^2 t\sin^2 t)^2}, \quad \text{at } t=0,\ \frac{d^2w}{dt^2}=0$$

2. Evaluate the iterated integrals (a) $\int_1^2\int_0^4 (x^2 - 2y^2)dxdy$ (10 points); (b) $\int_1^3\int_0^y \frac{4}{x^2+y^2}dxdy$. (10 points)

(a) $\int_1^2 \left[\frac{1}{3}x^3 - 2y^2x\right]_0^4 dy$

$= \int_1^2 \left(\frac{64}{3} - 8y^2\right)dy$

$= \left[\frac{64}{3}y - \frac{8}{3}y^3\right]_1^2$

$= \frac{8}{3}$

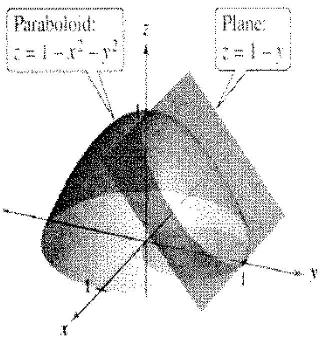
(b) $\int_1^3 \left[\frac{4}{y}\arctan\frac{x}{y}\right]_0^y dy$

$= \int_1^3 \left(\frac{4}{y}\arctan 1 - \frac{4}{y}\arctan 0\right)dy$

$= \int_1^3 \frac{\pi}{y}dy = \pi\left[\ln|y|\right]_1^3$

$= \pi(\ln 3 - \ln 1) = \pi\ln 3$

3. Find the volume of the solid region bounded above by the paraboloid $z = 1 - x^2 - y^2$ and below by the plane $z = 1 - y$. (16 points)



$1 - y = 1 - x^2 - y^2 \Rightarrow x^2 = y - y^2$

on xy-plane, $R: -\sqrt{y-y^2} \le x \le \sqrt{y-y^2}$

$0 \le y \le 1$

$V = \int_0^1\int_{-\sqrt{y-y^2}}^{\sqrt{y-y^2}} (1 - x^2 - y^2)\,dxdy - \int_0^1\int_{-\sqrt{y-y^2}}^{\sqrt{y-y^2}} (1-y)\,dxdy$

$= \int_0^1\int_{-\sqrt{y-y^2}}^{\sqrt{y-y^2}} (y - y^2 - x^2)\,dxdy$

$= \int_0^1 \left[(y-y^2)x - \frac{x^3}{3}\right]_{-\sqrt{y-y^2}}^{\sqrt{y-y^2}} dy = \frac{4}{3}\int_0^1 (y-y^2)^{\frac{3}{2}}dy$

$= \frac{4}{3}\cdot\frac{1}{8}\int_0^1 \left[1-(2y-1)^2\right]^{\frac{3}{2}}dy$ 令 $2y - 1 = 1\cdot\sin\theta$

$= \frac{1}{6}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^4\theta}{2}d\theta = \frac{1}{6}\cdot\frac{1}{2}\cdot 2\int_0^{\frac{\pi}{2}} \cos^4\theta\, d\theta$ — use Wallis's formula

$= \frac{1}{6}\cdot\frac{3\pi}{16} = \frac{\pi}{32}$