國立宜蘭大學 104 學年度第 1 學期 期末 考試試題紙			第 頁
考試科目	班級	學號	姓名
微積分一			

1. Find the average value of $f(x) = 3x^2 - 2x$ on the interval [1 , 4]. (10 points)

$\frac{1}{4-1}\int_1^4 (3x^2-2x)dx = \frac{1}{3}\left[x^3-x^2\right]_1^4 = \frac{1}{3}\left[(64-16)-(1-1)\right] = 16$

2. Find the indefinite integral. (30 points)

(a) $\int \frac{1}{\theta^2}\cos\frac{1}{\theta}d\theta$

(b) $\int(\sec t + \tan t)dt$

(c) $\int \frac{\cosh x}{\sinh x}dx$

(a) assume $u = \frac{1}{\theta}$

$du = -\frac{1}{\theta^2}d\theta$

原式 $= \int -\cos u\,du$

$= -\int \cos u\,du$

$= -\sin u + C$

$= -\sin\frac{1}{\theta} + C$

(b) $\int(\sec t+\tan t)dt$

$= \ln|\sec t+\tan t| - \ln|\cos t| + C$

(c) assume $u = \sinh x$, $du = \cosh x\,dx$

原式 $= \int \frac{1}{u}du = \ln|u| + C$

$= \ln|\sinh x| + C$

3. Find the definite integral. (30 points)

(a) $\int_1^9 \frac{1}{\sqrt{x}(1+\sqrt{x})^2}dx$

(b) $\int_0^{\ln 2} 2e^{-x}\cosh x\,dx$

(c) $\int_0^{1/\sqrt{2}} \frac{\arccos x}{\sqrt{1-x^2}}dx$

(a) assume $u = 1+\sqrt{x}$, $du = \frac{1}{2}\frac{1}{\sqrt{x}}dx$, $2du = \frac{1}{\sqrt{x}}dx$

$x=1, u=2$

$x=9, u=4$, $\int_2^4 \frac{2}{u^2}du = \left[2(-1)\cdot u^{-1}\right]_2^4 = -\frac{2}{4}+\frac{2}{2} = \frac{1}{2}$

(b) $2e^{-x}\cosh x = 2\cdot e^{-x}\left(\frac{e^x+e^{-x}}{2}\right) = 1+e^{-2x}$

$\int_0^{\ln 2} 2e^{-x}\cosh x\,dx = \int_0^{\ln 2}(1+e^{-2x})dx = \left\{x-\frac{1}{2}e^{-2x}\right]_0^{\ln 2} = \frac{3}{8}+\ln 2$

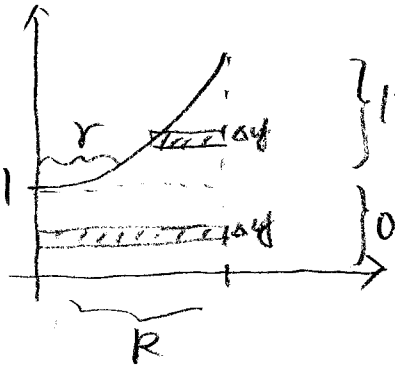
(c) assume $u = \arccos x$

$du = -\frac{1}{\sqrt{1-x^2}}dx$, $x=0, u=\frac{\pi}{2}$

$x=\frac{1}{\sqrt{2}}, u=\frac{\pi}{4}$

$\Rightarrow -\int_{\frac{\pi}{2}}^{\frac{\pi}{4}} u\,du = -\frac{1}{2}u^2\Big|_{\frac{\pi}{2}}^{\frac{\pi}{4}} = \frac{3\pi^2}{32}$

4. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^2+1$, $y=0$ $x=0$, and $x=1$ about the y-axis. (Using the Disk Method) (20 points)



$1\le y\le 2, R=1, r=\sqrt{y-1}$

$0\le y\le 1, R=1, r=0$

$V = \pi\int_0^1 (1^2-0^2)dy + \pi\int_1^2 \{1^2-(\sqrt{y-1}$

$= \pi\int_0^1 1\cdot dy + \pi\int_1^2 (2-y)dy$

$= \frac{3\pi}{2}$

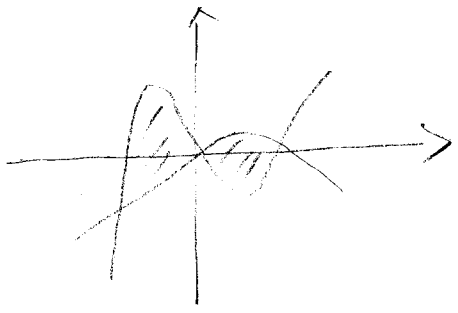
5. Find the area of the region between the graphs of $f(x) = 3x^3 - x^2 - 10x$ and $g(x) = -x^2+2x$.(10 points)

$3x^3-x^2-10x = -x^2+2x$

$3x^3-12x=0$

$3x(x+2)(x-2)=0$

$x=-2, 0, 2$



$A = \int_{-2}^0 [3x^3-x^2-10x-(-x^2+2x)]dx +$

$\int_0^2 [-x^2+2x-(3x^3-x^2-10x)]dx$

$= \int_{-2}^0 (3x^3-12x)dx + \int_0^2 (-3x^3+12x)dx$

$= \left[\frac{3}{4}x^4-6x^2\right]_{-2}^0 + \left[-\frac{3}{4}x^4+6x^2\right]_0^2$

$= 24$