國立宜蘭大學 104 學年度第 2 學期 期末 考試試題紙			第 頁
考試科目	班級	學號	姓名
微積分二			

Multiple choices (60 points)

1. (a) Find the radius of convergence of the power series $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$. (a) ∞ (b) 1 (c) 2 (d) 3

2. (a) Find the interval of convergence of the power series $\sum_{n=1}^{\infty}(-1)^{n+1}(n+1)x^n$. (a) $(-1,1)$ (b) $(-1,1]$ (c) $[-1,1)$ (d) $[-1,1]$

3. (d) Consider the function given by $f(x)=\sum_{n=1}^{\infty}\frac{x^n}{n}$. Find the intervals of convergence for $\int f(x)dx$. (a) $(-1,1)$ (b) $(-1,1]$ (c) $[-1,1)$ (d) $[-1,1]$

4. (c) Find a power series for $f(x)=\frac{4}{x+2}$, centered at 0. (a) $\sum_{n=0}^{\infty}4(-\frac{x}{2})^n$ (b) $\sum_{n=0}^{\infty}(-\frac{x}{2})^n$ (c) $\sum_{n=0}^{\infty}2(-\frac{x}{2})^n$ (d) $\sum_{n=0}^{\infty}(-x)^n$

5. (b) Find fx and fy and each at $(1, \ln 2)$ for $f(x,y)=xe^{x^2y}$. (a) $f_x = 4\ln 2,\ f_y = 2$ (b) $f_x = 4\ln 2 + 2,\ f_y = 2$ (c) $f_x = 4\ln 2 + 1,\ f_y = 2$ (d) $f_x = \ln 2 + 2,\ f_y = 2$

6. (d) Find $\frac{\partial w}{\partial s}$ for $w = 2xy$ where $x = s^2 + t^2$ and $y = s/t$. (a) $\frac{6s+2t}{t}$ (b) $\frac{6s+2t}{t^2}$ (c) $\frac{6s^2+2t^2}{t^2}$ (d) $\frac{6s^2+2t^2}{t}$

7. (c) Find $\frac{\partial w}{\partial t}$ when $s=1$ and $t=2\pi$ for $w = xy + yz + xz$ where $x = s\cos t$, $y = s\sin t$, and $z = t$. (a) 1 (b) 2 (c) $2+2\pi$ (d) 2π

8. (a) Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial \theta}$ for $w=\arctan\frac{y}{x}$ where $x = r\cos\theta$ and $y = r\sin\theta$. (a) $\frac{\partial w}{\partial r}=0, \frac{\partial w}{\partial \theta}=1$ (b) $\frac{\partial w}{\partial r}=1, \frac{\partial w}{\partial \theta}=0$ (c) $\frac{\partial w}{\partial r}=0, \frac{\partial w}{\partial \theta}=0$ (d) $\frac{\partial w}{\partial r}=1, \frac{\partial w}{\partial \theta}=1$

9. (d) Find $\frac{d^2w}{dt^2}$ when $t=0$ for $w=\arctan(2xy)$ where $x=\cos t$, $y=\sin t$. (a) 1 (b) 2 (c) $\frac{1}{\sqrt{2}}$ (d) 0

10. (b) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ when $xz + yz + xy = 0$ (a) $\frac{\partial z}{\partial x} = -\frac{x+z}{x+y}, \frac{\partial z}{\partial y} = -\frac{y+z}{x+y}$ (b)

$\frac{\partial z}{\partial x} = -\frac{y+z}{x+y}, \frac{\partial z}{\partial y} = -\frac{x+z}{x+y}$ (c) $\frac{\partial z}{\partial x} = -\frac{x}{z}, \frac{\partial z}{\partial y} = -\frac{y}{z}$ (d) $\frac{\partial z}{\partial x} = -\frac{y}{z}, \frac{\partial z}{\partial y} = -\frac{x}{z}$

Calculation

1. Evaluate the iterated integrals (a) $\int_0^2\int_{y^2}^4 dxdy$, (b) $\int_0^{\pi/2}\int_0^1 y\cos x dydx$, and (c) $\int_0^{\ln 4}\int_0^{\ln 3} e^{x+y}dydx$. (24 points)

(a) $\int_0^2\int_{y^2}^4 dxdy = \int_0^2 x\Big]_{y^2}^4 dy = \int_0^2 (4-y^2)dy = \frac{16}{3}$

(b) $\int_0^{\frac{\pi}{2}}\int_0^1 y\cos x\,dy\,dx = \int_0^{\frac{\pi}{2}} \frac{1}{2}[\cos x]y^2\Big]_0^1 dx = \int_0^{\frac{\pi}{2}} \frac{1}{2}\cos x\,dx$

$= \left[\frac{1}{2}\sin x\right]_0^{\frac{\pi}{2}} = \frac{1}{2}\left(\sin\frac{\pi}{2} - \sin 0\right) = \frac{1}{2}$

(c) $\int_0^{\ln 4}\int_0^{\ln 3} e^{x+y}dydx = \int_0^{\ln 4}\int_0^{\ln 3} e^x e^y dydx = \int_0^{\ln 4} e^x e^y\Big]_0^{\ln 3} dx$

$= \int_0^{\ln 4} e^x(e^{\ln 3} - e^0)dx = \int_0^{\ln 4} 2e^x dx = 2e^x\Big]_0^{\ln 4} = 2(e^{\ln 4} - e^0)$

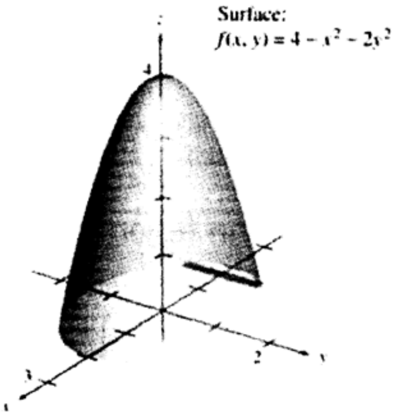
$= 6$

2. Find the volume of the solid region bounded by the paraboloid $z = 4 - x^2 - 2y^2$ and the xy-plane. (16 points)

for y: $-\sqrt{\frac{4-x^2}{2}} \le y \le \sqrt{\frac{4-x^2}{2}}$

when $z=0$, $x^2+2y^2=4$ in xy-plane

for x: $-2 \le x \le 2$



$V = \int_{-2}^{2}\int_{-\sqrt{\frac{4-x^2}{2}}}^{\sqrt{\frac{4-x^2}{2}}} (4-x^2-2y^2)\,dy\,dx$

$= \int_{-2}^{2}\left[(4-x^2)y - \frac{2y^3}{3}\right]_{-\sqrt{\frac{4-x^2}{2}}}^{\sqrt{\frac{4-x^2}{2}}} dx = \frac{4}{3\sqrt{2}}\int_{-2}^{2}(4-x^2)^{3/2}dx$

三角代換 assume $x = 2\sin\theta$, $x=2$, $\theta = \frac{\pi}{2}$; $x=-2$, $\theta = -\frac{\pi}{2}$

$= \frac{4}{3\sqrt{2}}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 16\cos^4\theta\,d\theta = \frac{64}{3\sqrt{2}}\cdot 2\int_0^{\frac{\pi}{2}}\cos^4\theta\,d\theta$

Wallis's formula $= \frac{128}{3\sqrt{2}}\cdot\left(\frac{3\pi}{16}\right) = 4\sqrt{2}\,\pi$