| 國立宜蘭大學　104 學年度第 1 學期 期中 考試試題紙 | | | 第　　頁 |
|---|---|---|---|
| 考　試　科　目 | 班　　　　級 | 學　　　　號 | 姓　　　　名 |
| Calculus | | | |

1. (20%) Apply Rolle's Theorem to $f(x) = \sin x$ on the closed interval $[0, 2\pi]$ for determining a value $c$ such that $f'(c) = 0$.

$f'(x) = \cos x$

$\cos x = 0$, $x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\therefore c = \frac{\pi}{2}, \frac{3\pi}{2}$

2. (20%)Find the derivative of the function.

(a) $f(x) = e^x \arcsin x$　　(b) $e^{xy} + x^2 - y^2 = 10$

(a) $f'(x) = e^x \cdot \arcsin x + e^x \cdot \frac{1}{\sqrt{1-x^2}}$

(b) $e^{xy} \cdot (1 \cdot y + x \cdot \frac{dy}{dx}) + 2x - 2y \cdot \frac{dy}{dx} = 0$

$y \cdot e^{xy} + xe^{xy} \cdot \frac{dy}{dx} + 2x - 2y\frac{dy}{dx} = 0$

$(x \cdot e^{xy} - 2y)\frac{dy}{dx} = -(2x + y \cdot e^{xy})$

$\frac{dy}{dx} = \frac{-(2x + y \cdot e^{xy})}{xe^{xy} - 2y}$

3. (20%) Find an equation of the tangent line to the graph of the equation at the given point (1,0)

$\arctan(x + y) = y^2 + \frac{\pi}{4}$.

$\frac{1}{1+(x+y)^2} \cdot (1 + \frac{dy}{dx}) = 2y \cdot \frac{dy}{dx}$

$\frac{1}{1+(x+y)^2} + \frac{1}{1+(x+y)^2}\frac{dy}{dx} = 2y\frac{dy}{dx}$

$\Rightarrow (2y - \frac{1}{1+(x+y)^2})\frac{dy}{dx} = \frac{1}{1+(x+y)^2}$

$\therefore -\frac{1}{1+1^2}\frac{dy}{dx} = \frac{1}{1+1^2}$, $\therefore \frac{dy}{dx} = -1$

tangent line:

$y - 0 = -1(x-1) \Rightarrow y = -x + 1$

4. (20%)Determine the following limits.

(a) $\lim_{x\to 0} \frac{4(e^{2x}-1)}{e^x - 1}$　　(b) $\lim_{x\to \pi/4} \frac{1-\tan x}{\sin x - \cos x}$

(a) 原式 $= \lim_{x\to 0} \frac{4(e^x+1)(e^x-1)}{e^x - 1}$

$= \lim_{x\to 0} 4(e^x+1) = 4 \times 2$

$= 8$

(b) 原式 $= \lim_{x\to \frac{\pi}{4}} \frac{1 - \frac{\sin x}{\cos x}}{\sin x - \cos x} = \lim_{x\to \frac{\pi}{4}} \frac{\frac{\cos x - \sin x}{\cos x}}{\sin x - \cos x}$

$= \lim_{x\to \frac{\pi}{4}} \frac{-(\sin x - \cos x)}{\cos x(\sin x - \cos x)} = \lim_{x\to \frac{\pi}{4}} \frac{-1}{\cos x} = \frac{-1}{\cos\frac{\pi}{4}} = -\sqrt{2}$

5. (20%) Find the critical numbers of $f(x)$..

(a) $f(x) = \frac{x^4+1}{x^2}$　　(b) $f(x) = (x-1)e^x$

(a) $f(x) = x^2 + x^{-2}$

$f'(x) = 2x - 2x^{-3}$

$= \frac{2(x^4-1)}{x^3} = \frac{2(x^2+1)(x-1)(x+1)}{x^3}$

$f'(x) = 0$, $x = \pm 1$; $f'(x)$ 不存在, $x = 0$

$\therefore$ 臨界數 $x = 0, -1, 1$

(b) $f'(x) = e^x + (x-1)e^x$

$= e^x + xe^x - e^x = xe^x$

$f'(x) = 0$, $x = 0$

臨界數 $x = 0$