國立宜蘭大學 104 學年度第 2 學期 期中 考試試題紙			第 頁
考試科目	班級	學號	姓名
微積分二			

Multiple choices (60%)

1. ( ) Find the area of the surface formed by revolving the graph of $f(x)=\frac{x^3}{6}+\frac{1}{2x}$ on the interval [1 , 2] about the x-axis. (a) ~~2π~~ (b) 3π (c) ~~4π~~ (d) ~~6π~~.

2. ( ) Solve the arc length of the graph of $y=\ln\cos x$ on the interval $[0,\pi/3]$. (a) $\ln(2+\sqrt{3})$ (b) $2\ln 3$ (c) $\ln 3$ (d) $\ln(3+\sqrt{2})$.

3. ( ) Solve the indefinite integral of $\int \sec^3 x dx$. (a) $\frac{1}{2}\sec x\tan x+\frac{1}{3}\ln|\sec x+\tan x|+C$ (b) $\frac{1}{2}\sec x\tan x+\frac{1}{3}\ln|\sec x^2+\tan x|+C$ (c) $\frac{1}{2}\sec x\tan x-\frac{1}{2}\ln|\sec x+\tan x|+C$ (d) $\frac{1}{2}\sec x\tan x+\frac{1}{2}\ln|\sec x+\tan x|+C$.

4. ( ) Solve the indefinite integral of $\int x^2\cos x dx$. (a) $x^2\sin x+2x\cos x-2\sin x+C$ (b) $x^2\cos x+2x\cos x-2\sin x+C$ (c) $x^2\sin x+2x\sin x-2\sin x+C$ (d) $x^2\sin x+2x\cos x-2\cos x+C$

5. ( ) Solve the indefinite integral of $\int \sin(-4x)\cos 3x dx$. (a) $\frac{\cos x}{7}+\frac{\cos 7x}{14}+C$ (b) $\frac{\cos x}{14}+\frac{\cos 7x}{7}+C$ (c) $\frac{1}{14}(\cos x+7\cos 7x)+C$ (d) $\frac{1}{14}(7\cos x+\cos 7x)+C$.

6. ( ) Solve the indefinite integral of $\int \frac{2x^3-4x-8}{(x^2-x)(x^2+4)}dx$. (a) $\ln|x|-\ln(x^2+4)+2\arctan\frac{x}{2}+C$ (b) $2\ln|x|-2\ln|x-1|+\ln(x^2+4)+2\arctan\frac{x}{2}+C$ (c) $\ln|x|-\ln|x-1|+2\ln(x^2+4)+C$ (d) $-2\ln|x-1|+\ln(x^2+4)+\arctan\frac{x}{2}+C$

7. ( ) Solve the definite integral of $\int_0^{\pi/4}\sec^2 t\sqrt{\tan t}dt$. (a) $2/3$ (b) 1 (c) $1/2$ (d) $1/4$.

8. ( ) Solve the definite integral of $\int_0^{\sqrt{3}/2}\frac{t^2}{(1-t^2)^{3/2}}dt$. (a) $\sqrt{3}+\pi/2$ (b) $\sqrt{3}-\pi/3$ (c) $\sqrt{3}\pi/3$ (d) $\pi/2$.

9. ( ) Solve the definite integral of $\int_{\sqrt{3}}^{2}\frac{\sqrt{x^2-3}}{x}dx$. (a) $1-\sqrt{3}\pi/6$ (b) $\sqrt{3}-\pi/3$ (c) $\sqrt{3}\pi/3$ (d) $\pi/2-1$.

10. ( ) Solve the definite integral of $\int_2^3\frac{2x-3}{(x-1)^2}dx$. (a) $2\ln 2-1/2$ (b) $2\ln 2$ (c) $2\ln 3$ (d) $\ln 3-1/2$.

Calculation (40%)

1. Evaluate the following limits: (20%)

(a) $\lim_{x\to 0}\frac{\arctan x}{\sin x}$ L'Hôpital

$$=\lim_{x\to 0}\frac{\frac{1}{x^2+1}}{\cos x}$$

$$=1$$

(b) $\lim_{x\to 1^+}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)$

$$=\lim_{x\to 1^+}\left[\frac{x-1-\ln x}{(x-1)\ln x}\right]$$ L'Hôpital rule

$$=\lim_{x\to 1^+}\frac{1-\frac{1}{x}}{\ln x+(x-1)\left(\frac{1}{x}\right)}$$

$$=\lim_{x\to 1^+}\frac{x-1}{x-1+x\ln x}$$ L'Hôpital rule

$$=\lim_{x\to 1^+}\frac{1}{1+\ln x+1}=\frac{1}{2}$$

2. Find the improper integral. (20%)

(a) $\int_{-1}^{2}\frac{1}{x^3}dx$

$$=\int_{-1}^{0}\frac{1}{x^3}dx+\int_{0}^{2}\frac{1}{x^3}dx$$

$$=\lim_{b\to 0^-}\int_{-1}^{b}\frac{1}{x^3}dx+\lim_{a\to 0^+}\int_{a}^{2}\frac{1}{x^3}dx$$

$$=\lim_{b\to 0^-}\left[-2x^{-2}\right]_{-1}^{b}+\lim_{a\to 0^+}\left[-2x^{-2}\right]_{a}^{2}$$

$$=\infty$$

(b) $\int_{0}^{\infty}\frac{1}{\sqrt{x}(x+1)}dx$

$$=\int_{0}^{1}\frac{dx}{\sqrt{x}(x+1)}+\int_{1}^{\infty}\frac{dx}{\sqrt{x}(x+1)}$$ [assume $u=\sqrt{x}$, $du=\frac{1}{2}\frac{1}{\sqrt{x}}dx$]

$$=\lim_{b\to 0^+}\int_{b}^{1}\frac{2}{u^2+1^2}du+\lim_{c\to\infty}\int_{1}^{c}\frac{2}{u^2+1^2}du$$

$$=\lim_{b\to 0^+}\left[2\arctan u\right]_{b}^{1}+\lim_{c\to\infty}\left[2\arctan u\right]_{1}^{c}$$

$$=2\left[\left(\frac{\pi}{4}\right)-0\right]+2\left[\frac{\pi}{2}-\frac{\pi}{4}\right]=\pi$$