Ch 17 Spontaneity of Reaction

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The goal of this chapter is to answer a basic question: Will a given reaction occur "by itself" at a particular temperature and pressure, without exertion of any outside force?

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Is the reaction spontaneous?

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To develop a general criterion for spontaneity, we will apply the principles of thermodynamics, the science that deals with heat and energy effects.

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Three different thermodynamic functions are of value in analyzing spontaneity.

- 1. ΔH , the change in enthalpy; a negative value of ΔH tends to make a reaction spontaneous.
- 2. ΔS , the change in entropy; a positive value of ΔS tends to make a reaction spontaneous.
- 3. ΔG , the change in free energy, a reaction at constant temperature and pressure will be spontaneous if ΔG is negative, no ifs, ands, or buts.

The free energy change, ΔG , can be also used to:

- **g** Determine the effect of temperature, pressure, and concentration on reaction spontaneity. (Ch. 17-5)
- g Calculate the equilibrium constant for a reaction. (Ch. 17-6)
- g Determine whether coupled reaction will be spontaneous. (Ch. 17-7)

§ 17-1 Spontaneous Processes

Spontaneous process; for example:

g An ice cube melts when added to a glass of water at room temperature.

$$H_2O(s) \longrightarrow H_2O(\mathbf{l})$$

g A mixture of hydrogen and oxygen burns if ignited by a spark.

$$2H_2(g) + O_2(g) \longrightarrow 2H_2O(\mathbf{l})$$

g An iron (steel) tool exposed to moist air rusts.

$$2Fe(s) + \frac{3}{2}O_2(g) + 3H_2O(\mathbf{l}) \longrightarrow 2Fe(OH)_3(s)$$

These three reactions are spontaneous at 25 $^{\circ}$ C and 1 atm.

 $\downarrow \downarrow$

The word "spontaneous" does not imply anything about how rapidly a reaction occurs. ⇒ reaction rate.

If a reaction is spontaneous under a given set of conditions, the reverse reaction must be nonspontaneous.

$$2H_2O(\mathbf{1}) \longrightarrow 2H_2(g) + \mathcal{O}_2(g)$$
 nonspontaneous

However, it is often possible to bring about a nonspontaneous reaction by supplying energy in the form of work.

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Electrolysis can be used to decompose water to elements.

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To do this, electrical energy must be furnished, perhaps from a storage battery.

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A spontaneous process is one that moves the reaction toward equilibrium. A nonspontaneous process is moves the system away from equilibrium.

§ The energy factor $\Rightarrow \Delta H$

Many spontaneous process proceed with a decrease of energy.

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One might guess that spontaneous reactions would be exothermic $(\Delta H < 0)$. \Rightarrow proposed by P.M. Berthelot.

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Nearly all exothermic chemical reactions are spontaneous at 25 °C and 1 atm. For example:

$$2H_{2}(g) + O_{2}(g) \longrightarrow 2H_{2}O(\mathbf{l})$$

$$2Fe(s) + \frac{3}{2}O_{2}(g) + 2H_{2}O(\mathbf{l}) + \longrightarrow 2Fe(OH)_{3}(s)$$

$$\downarrow \qquad \Delta H = -571.6 \text{ kJ}$$

$$\Delta H = -788.6 \text{ kJ}$$

On the other hand, the simple rule fails for many familiar phase changes.

$$H_2O(s) \longrightarrow H_2O(\mathbf{l})$$
 $\Delta H = +6.0 \text{ kJ}$

The process is not an exothermic, but it takes place spontaneously at 1atm above 0 $^{\circ}\text{C}$.

Endothermic reactions that are nonspontaneous at room temperature often become spontaneous when the temperature is raised. For example, the decomposition of the limestone.

$$\begin{array}{ccc} CaCO_{3}(s) & \xrightarrow{25\ ^{o}C,\ 1\ \text{atm}} & CaO(s) + CO_{2}(g) & \Delta H = +178.3\ \text{kJ} & \text{nonspontaneous} \\ & \xrightarrow{1100\ \text{K},\ 1\ \text{atm}} & CaO(s) + CO_{2}(g) & \Delta H \ ; \ +178.3\ \text{kJ} & \text{spontaneous} \\ & & & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & & \\ & & & & & & & & & & & & & \\ & & & & & & & & & & & & \\ & & & & & & & & & & & & \\ & & & & & & & & & & & & \\ & & & & & & & & & & & \\ & & & & & & & & & & & \\ & & & & & & & & & & & \\ & & & & & & & & & & & \\ & & & & & & & & & & \\ & & & & & & & & & & \\ & & & & & & & & & & \\ & & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & \\ & & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & &$$

The endothermic reactions becomes spontaneous at high temperatures, despite that ΔH is positive.

 ΔH is nearly independent of temperature (for this reaction).

§ The randomness factor $\Rightarrow \Delta S$

The direction of a spontaneous change is not always determined by the tendency for a system to go to a state of lower energy.

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Nature tends to move spontaneously from a state of lower probability to one of higher probability.

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Fig. 17-2 the system has gone from a highly ordered state (all the H_2 molecules on the left, all the N_2 molecules on the right) to a more disordered, or random state in which the molecules are distributed evenly between two bulbs.

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In general, nature tends to move spontaneously from more ordered to

more random states.

§ 17-2 Entropy, S

Entropy is often described as a measure of disorder or randomness.

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Boltzmann Relationships:

The absolute entropy S of the system is proportional to the natural logarithm of the possible combinations.

$$S = k \ln \Omega$$

 Ω : the number of possible *combinations*.

k: proportional constant, "Boltzmann constant"

$$k = \frac{R}{N_a}$$
; 1.381×10⁻²³ J/K

An increase in the number of possible microstates (i.e., disorder) results in an increase of entropy.

1

In general, the more random the state, the larger the number of its possible microstates, the more probable the state, thus the greater its entropy.

Entropy is a state property. That is, the entropy depends only on the state of a system, not on its history. The entropy change is determined by the entropies of the final and the initial states, not on the path followed from one state to another.

$$\Delta S = S_{final} - S_{initial}$$

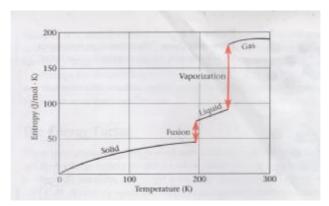


Fig. 17-4: Molar entropy of ammonia as a function of temperature.

Several factors influence the amount of entropy:

g A liquid has a higher entropy than the solid from which it is formed.

$$S(\mathbf{l}) > S(s)$$

- $S(\mathbf{l})$: molecules can occupy many more positions as they move away from the lattice.
- S(s): molecules are confined to fixed position.
- **g** A gas has a higher entropy than the liquid from which it is formed. S(g) > S(1)
- **g** Increasing the temperature of a substance increase its entropy.

$$T \uparrow \Rightarrow S \uparrow$$

The third law of thermodynamics:

Absolute entropy approaches zero as the absolute temperature approaches zero.

- **g** Melting and vaporization are accompanied by relatively large increase in entropy.
- Ex. 17.1: Predict whether ΔS is positive or negative for each of the following processes:
 - a). taking dry ice from a freezer where its temperature is -80 $^{\circ}$ C and allowing it to warm to room temperature.
 - b). dissolving bromine in hexane.
 - c). condensing gaseous bromine to liquid bromine.

Ans:

a).
$$T \uparrow \Rightarrow S \uparrow \therefore \Delta S \uparrow$$

b). **Q** more random $S \uparrow \Rightarrow \Delta S > 0$

c).
$$S(g) > S(1)$$
 :: $g \rightarrow 1$ $\Delta S \downarrow \Rightarrow \Delta S < 0$

§ Standard molar entropies, So

The entropy of a substance, unlike its enthalpy, can be evaluated directly.

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The only state property which has the absolute value.

- g elements have nonzero standard entropies.
- **g** standard molar entropies of pure substance (elements and compounds) are always positive quantities. $(S^o > 0)$
- **g** aqueous ions may have negative values. Ex: $S_{F^{-}(aa)}^{o} = -13.8 \text{ J/mol} \cdot \text{K}$

Defined:
$$S_{H^+(aq)}^o = 0$$

Among substances of similar structure and physical state, entropy usually increases with molar mass.

$$CH_4(g) \qquad S^o = 186.2 \text{ J/mol} \cdot \text{K}$$
 Ex:
$$C_2H_6(g) \qquad S^o = 229.5 \text{ J/mol} \cdot \text{K}$$

$$C_3H_8(g) \qquad S^o = 269.9 \text{ J/mol} \cdot \text{K}$$

§ ΔS^{o} for reactions

 ΔS^{o} , the standard entropy change, for reactions can be determined as:

$$\Delta S^o = \sum S^o_{products} - \sum S^o_{reactants}$$
 (17-1)

Ex: Consider the reaction, calculate ΔS^{o} :

$$CaCO_{3}(s) \longrightarrow CaO(s) + CO_{2}(g)$$

$$\Delta S^{o} = \sum S^{o}_{products} - \sum S^{o}_{reactants}$$

$$= [1 \cdot S^{o}_{CaO(S)} + 1 \cdot S^{o}_{CO_{2}(g)}] - (1 \cdot S^{o}_{CaCO_{3}(S)})$$

$$= [1 \cdot 39.8 + 1 \cdot 213.6] - (1 \cdot 92.9)$$

$$= +160.5 \text{ J/K}$$

$$\mathbf{Q} \quad \Delta \mathbf{n}_{g} = 1 - 0 = 1 \quad \therefore \Delta S^{o} > 0$$

A reaction that results in an increase in the number of moles of gas is accompanied by an increase in entropy. Conversely, if the number of moles of gas decreases, ΔS^o is a negative quantity.

Ex.

$$\begin{split} 2H_2(g) + \mathrm{O}_2(g) &\longrightarrow 2\mathrm{H}_2O(\mathbf{l}) \\ \Delta S^o &= \sum S^o_{products} - \sum S^o_{reac\, tants} \\ &= (2 \cdot S^o_{H_2O(\mathbf{l})}) - [2 \cdot S^o_{H_2(g)} + 1 \cdot S^o_{O_2(g)}] \\ &= (2 \cdot 69.9) - [2 \cdot (130.6) + 1 \cdot (205.0)] \\ &= -326.4 \, \mathrm{J/K} \\ \Delta \mathrm{n}_g &= 0 - (2 + 1) = -3 \quad "-" \implies \Delta S^o < 0 \end{split}$$

Ex. 17.2: Calculate ΔS^o for

- a). dissolving one mole of calcium hydroxide in water.
- b). the combustion of one gram of methane to form carbon dioxide and liquid water.

Ans:

a).
$$\operatorname{Ca}(\operatorname{OH})_2(s) \longrightarrow \operatorname{Ca}^{2+}(aq) + 2\operatorname{OH}^-(aq)$$

$$\Delta S^o : +83.4 \qquad -53.1 \qquad -10.8 \text{ J/K}$$

$$\Delta S^o = \begin{bmatrix} 1 \cdot S^o_{Ca^{2+}(aq)} + 2 \cdot S^o_{OH^-(aq)} \end{bmatrix} - (1 \cdot S^o_{Ca(OH)_2(aq)})$$

$$= \begin{bmatrix} 1 \cdot (-53.1) + 2 \cdot (-10.8) \end{bmatrix} - (1 \cdot 83.40)$$

$$= -158.1 \text{ J/K} < 0 \qquad \therefore \text{ nonspontaneous}$$
b). $\operatorname{CH}_4(g) + 2\operatorname{O}_2(g) \longrightarrow \operatorname{CO}_2(g) + 2\operatorname{H}_2O(\mathbf{l})$

$$\Delta S^o : +186.2 \quad +205.0 \qquad +213.6 \qquad +69.9$$

$$\Delta S^o = \begin{bmatrix} 1 \cdot S^o_{CO_2(g)} + 2 \cdot S^o_{H_2O(1)} \end{bmatrix} - \begin{bmatrix} 1 \cdot S^o_{CH_4(g)} + 2 \cdot S^o_{O_2(g)} \end{bmatrix}$$

$$= \begin{bmatrix} 1 \cdot 213.6 + 2 \cdot 69.9 \end{bmatrix} - \begin{bmatrix} 1 \cdot 186.2 + 2 \cdot 205.0 \end{bmatrix}$$

$$= -242.8 \text{ J/K} \text{ for the combustion of one mole of } \operatorname{CH}_4(g)$$
For the combustion of one gram of $\operatorname{CH}_4(g)$ is:
$$\frac{1.00}{16.04} \cdot (-242.8) = -15.14 \text{ J/K}$$

§ The second law of thermodynamics:

The relationship between entropy and spontaneity can be expressed through a basic principle of nature known as the second law of thermodynamics.

⇓

For an isolated system, if a spontaneous change occurs, it occurs with a concurrent increase in the entropy of the system.

$$\Delta S_{universe} = (\Delta S_{system} + \Delta S_{surroundings}) > 0$$
 spontaneous system

For many spontaneous processes, the entropy change for the system is negative quantity. Consider, for example, the rusting of iron, a spontaneous process:

$$2Fe(s) + \frac{3}{2}O_{2}(g) + 3H_{2}O(\mathbf{l}) \longrightarrow 2Fe(OH)_{3}(s)$$

$$\Delta S^{o}: 27.3 \quad 205.0 \quad 69.9 \quad 106.7$$

$$\Delta S^{o} = (2 \cdot 106.7) - [2 \cdot (27.3) + \frac{3}{2} \cdot (205.0) + 3 \cdot (69.9)]$$

$$= -358.4 \text{ J/K} < 0 \qquad \therefore \text{ nonspontaneous}$$

All the law requires is that the entropy change of the surroundings be greater than 358.4 J/K, so that $\Delta S_{universe} > 0$.

$$\Delta S_{universe} = (\Delta S_{system} + \Delta S_{surroundings}) > 0$$

$$= (-358.4) + \Delta S_{surroundings} > 0$$

$$\Rightarrow \Delta S_{surroundings} > 358.4 \text{ J/K}$$

The calculate the entropy change for the surroundings is not easy. \fill

We deals only with the thermodynamic properties of chemical systems.

§ 17-3 Free Energy, G

To put the enthalpy and the entropy together in such a way as to arrive at a single function whose sign will determine whether a reaction is spontaneous.

J. W. Gibbs introduced a new quantity.

Gibbs free energy, G.
$$G = H - TS$$

For a reaction taking place at constant pressure and temperature, ΔG represents that portion of the total energy change that is available (i.e., "free") to do useful work. For example, ΔG for a reaction is -270 kJ, it is possible to obtain 270 kJ of useful work from the reaction. Conversely, if ΔG is +270 kJ, at least that much energy in the form of work must be supplied to

make the reaction take place.

 $\begin{picture}(20,0) \put(0,0){\line(0,0){100}} \put(0,0){\line(0,0){100$

 ΔG for a reaction depends only on the nature of products and reactants and the conditions (temperature, pressure, and concentration).

- **g** If ΔG is negative, the reaction is spontaneous.
- **g** If ΔG is positive, the reaction will not take place spontaneously. Instead, the reverse reaction will be spontaneous.
- **g** If ΔG is 0, the system is at equilibrium; there is no tendency for reaction to occur in either direction.

 ΔG is a measure of the "driving force of a reaction". Reactions, at constant pressure and temperature, go in such a direction as to decrease the free energy of the system.

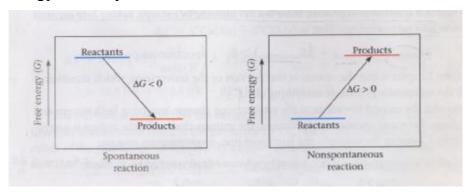


Fig. 17.5. Sign of free energy and spontaneity.

- **g** If the products at the specific conditions of temperature, pressure, and concentration have a lower free energy than the reactants ($G_{products} < G_{reactants} \Rightarrow \Delta G < 0$), the forward reaction will occur.
- **g** If the reverse is true (($G_{reactants} < G_{products} \implies \Delta G > 0$), the reverse reaction is spontaneous.
- g If $G_{products} = G_{reactants}$ ($\Rightarrow \Delta G = 0$) there is no driving force to make the reaction go in either direction.

§ Relation among ΔG , ΔH , and ΔS

Gibbs-Helmholtz equation: $\Delta G = \Delta H - T\Delta S$ at constant temperature.

Two factors tend to make ΔG negative and hence lead to a spontaneous reaction:

1. A negative value of ΔH :

Exothermic reaction ($\Delta H < 0$) tend to be spontaneous. On the molecule level, this means that there will be a tendency to form "strong" bonds at the expense of "weak" ones.

2. A positive value of ΔS :

 $\Delta S > 0 \Rightarrow -T\Delta S < 0$. Hence there will be a tendency for a reaction to be spontaneous if the products are less ordered than the reactants.

§ 17-4 Standard Free Energy Change, ΔG°

Standard conditions: $T = 25^{\circ}C$

Gas: partial pressure = 1 atm.

Molar concentration of ions or molecules in solution = 1 M.

$$\Delta G^{\circ} = \Delta H^{\circ} - T\Delta S^{\circ}$$

1. If ΔG° is negative, the reaction is spontaneous at standard conditions.

$$CaO(s) + CO_2(g, 1 \text{ atm}) \longrightarrow CaCO_3(s)$$

$$\Delta G^{\circ} = -130.4 \text{ kJ}$$

2. If ΔG° is positive, the reaction is nonspontaneous at standard conditions.

$$AgCl(s) \longrightarrow Ag^{+}(aq, 1M) + Cl^{-}(aq, 1M)$$

$$\Delta G^o = +55.7 \text{ kJ}$$

The reverse reaction is spontaneous.

3. If ΔG° is 0, the system is at equilibrium at standard conditions.

$$H_2O(\mathbf{l}) \longrightarrow H_2O(g, 1atm)$$

$$\Delta G^o = 0 \text{ kJ}$$

§ Calculation of ΔG° at 25 °C; free energy of formation

Ex. 17.3: Calcium sulfate, CaSO₄, is used as a drying agent and sold under trade name Drierite. For the reaction:

$$CaSO_4(s) \longrightarrow Ca^{2+}(aq) + SO_4^{2-}(aq)$$

Calculate *a*). ΔH^o b). ΔS^o c). ΔG^o at 25 oC .

Ans:

$$CaSO_{4}(s) \longrightarrow Ca^{2+}(aq) + SO_{4}^{2-}(aq)$$

$$\Delta H_{f}^{o}: -1434.1 \qquad -542.8 \qquad -909.3 \text{ kJ}$$

$$S^{o}: -106.7 \qquad -53.1 \qquad +20.1$$
a).
$$\Delta H^{o} = \sum \Delta H_{f,products}^{o} - \sum \Delta H_{f,reactants}^{o}$$

$$= [1 \cdot \Delta H_{f,Ca^{2+}(aq)}^{o} + 1 \cdot \Delta H_{f,SO_{4}^{2-}(aq)}^{o}] - (1 \cdot \Delta H_{f,CaSO_{4}(s)}^{o})$$

$$= [1 \cdot (-542.8) + 1 \cdot (-909.3)] - [1 \cdot (-1434.1)]$$

$$= -18.0 \text{ kJ}$$
b).
$$\Delta S^{o} = \sum S_{products}^{o} - \sum S_{reactants}^{o}$$

$$= [1 \cdot (-53.1) + 1 \cdot (+20.1)] - [1 \cdot (-106.7)]$$

$$= -139.7 \text{ J/K} = -0.1397 \text{ kJ/K}$$
c).
$$\Delta G^{o} = \Delta H^{o} - T\Delta S^{o}$$

$$= (-18.0) - 298 \cdot (-0.1397)$$

$$= +23.6 \text{ kJ}$$

§ Standard free energy of formation, ΔG_f^o

 ΔG_f^o is analogous to the enthalpy of formation, ΔH_f^o .

It is defined as the free energy per mole when a compound is formed from the elements in their stable states at 1 atm.

For most compounds, ΔG_f^o is a negative quantity, which means that the compound can be formed spontaneously from the elements. This is true for water:

$$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(\mathbf{l})$$
 $\Delta G_f^o = -237.2 \text{ kJ/mol}$

A few compounds, including acetylene, have positive free energies of formation ($\Delta G_{f,C_2H_2(1)}^o = +209.2 \text{ kJ/mol}$). These compounds cannot be made from the elements at ordinary temperatures and pressures; they are potentially unstable with respect to the elements. In the case of acetylene, the reaction:

$$C_2H_2(g) \longrightarrow 2C(s) + H_2(g)$$
 $\Delta G^o = -209.2 \text{ kJ}$

occurs with explosive violent unless special precaution are taken.

$$\Delta G_{reaction}^{o} = \sum \Delta G_{f,products}^{o} - \sum \Delta G_{f,reactants}^{o}$$
 (17-3)

 $\Delta G^o_{reaction}$ is valid only at the temperature at which ΔG^o_f data are tabulated, in this case 25 °C. ΔG^o varies considerably with temperature, so this approach is not even approximately valid at other temperature.

Ex. 17.4: Using ΔG_f^o values from Appendix 1, calculate the standard free energy change at 25°C for the reaction referred to in Example 17.3. Ans:

$$CaSO_{4}(s) \longrightarrow Ca^{2+}(aq) + SO_{4}^{2-}(aq)$$

$$\Delta G_{f}^{o}: -1321.8 \qquad -553.6 \qquad -744.5 \text{ kJ}$$

$$\Delta G^{o} = \sum \Delta G_{f,products}^{o} - \sum \Delta G_{f,reactants}^{o}$$

$$= [1 \cdot \Delta G_{f,Ca^{2+}(aq)}^{o} + 1 \cdot \Delta G_{f,SO_{4}^{2-}(aq)}^{o}] - (1 \cdot \Delta G_{f,CaSO_{4}(s)}^{o})$$

$$= [1 \cdot (-553.6) + 1 \cdot (-774.5)] - [1 \cdot (-1321.8)]$$

$$= +23.7 \text{ kJ} \approx \text{Ex. } 17.3: +23.6 \text{ kJ}$$

§ Calculation of ΔG^o at other temperature, ΔG_T

To a good degree of approximation, the temperature variation of ΔH^o and ΔS^o can be neglected. $\therefore \Delta H$ and ΔS always change in the same direction as the temperature changes.

$$\Delta G^{\circ} = \Delta H^{\circ} - T\Delta S^{\circ}$$

At temperatures other than 25°C, you need only change the value of T.

Ex. 17.5: Iron, a large component of steel, is obtained by reducing iron (III) oxide (present in hematite ore) with hydrogen in a blast furnace. Steam is a byproduct of the reaction. Calculate ΔG° at 230°C for the reduction of one mole of Fe₂O₃.

Ans:

Fe₂
$$O_3(s) + 3H_2(g) \longrightarrow 2\text{Fe(s)} + 3H_2O(g)$$

 ΔH_f^o : -824.2 0 0 -241.8 kJ
 S^o : 87.4 130.6 27.3 188.7 J/K

$$\Delta H^{o} = \sum \Delta H^{o}_{f,products} - \sum \Delta H^{o}_{f,reac tants}$$

$$= (3 \cdot \Delta H^{o}_{f,H_{2}O(1)}) - (1 \cdot \Delta H^{o}_{f,Fe_{2}O_{3}(s)})$$

$$= [3 \cdot (-241.8)] - [1 \cdot (-824.2)]$$

$$= +98.8 \text{ kJ}$$

$$\Delta S^{o} = \sum S^{o}_{products} - \sum S^{o}_{reac tants}$$

$$= [2 \cdot (27.3) + 3 \cdot (188.7)] - [1 \cdot (87.4) + 3 \cdot (130.6)]$$

$$= +141.5 \text{ J/K} = +0.1415 \text{ kJ/K}$$

$$\Delta G^{o} = \Delta H^{o} - T\Delta S^{o}$$

$$= (+98.8) - 503 \cdot (+0.1415)$$

$$= +27.6 \text{ kJ}$$

 ΔG° , unlike ΔH° and ΔS° , is strongly dependent on temperature. This comes about, of course, because of the T is in the Gibbs-Helmholtz equation:

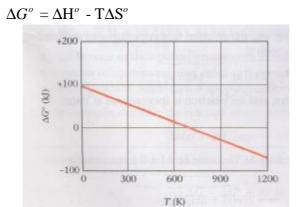


Fig. 17.6: Variation of ΔG° with T.

A plot of ΔG° versus T, should be linear, with a slope of $-\Delta S^{\circ}$ and a y-intercept (at 0 K) of ΔH° .

§ 17-5 Effect of Temperature, Pressure and

Concentration on Reaction Spontaneity

A change in reaction conditions can, and often does, change the direction in which a reaction occurs spontaneously.

$$\Delta G^o = \Delta H^o - T\Delta S^o$$

is readily applied to deduce the effect of temperature on reaction spontaneity.

§ Temperature

			-	<u> </u>
Case	ΔH^o	ΔS^o	$\Delta G^o = \Delta H^o - T\Delta S^o$	Remarks
I	-	+	always -	Spontaneous at all T; reverse reaction
				always nonspontaneous.
II	+	-	always +	Nonspontaneous at all T.
III	+	+	+ at low T	Nonspontaneous at low T, becomes
			- at high T	spontaneous at high T.
IV	-	-	- at low T	Spontaneous at low T, becomes
			+ at high T	nonspontaneous as T is raised.

Table 17-2: Effect of temperature on reaction spontaneity.

If ΔH^o and ΔS^o have opposite signs (case I and II), it is impossible to reverse the direction for spontaneity by a change in temperature alone.

Case III and IV, ΔH^o and ΔS^o have the same sign, at low temperatures, ΔH^o predominates, and the exothermic reaction, which may be either the forward or the reverse reaction, occurs. As the temperature raises, the quantity $T\Delta S^o$ increase in magnitude and eventually exceeds ΔH^o . At high temperatures, the reaction that leads to an increase in entropy occurs.

In most case, 25° C is a "low" temperature, at least at a pressure of 1 atm. This explains why exothermic reactions are usually spontaneous at room temperature and atmospheric pressure.

Ex. 17.6: At what temperature does ΔG° becomes zero for the reaction consider in Ex.17.5.

$$\operatorname{Fe_2O_3}(s) + 3\operatorname{H_2}(g) \longrightarrow 2\operatorname{Fe}(s) + 3\operatorname{H_2}O(\mathbf{l})$$

Ans:

$$\Delta G^{\circ} = \Delta H^{\circ} - T\Delta S^{\circ} = 0$$

$$T = \frac{\Delta H^{\circ}}{\Delta S^{\circ}} = \frac{98.8 \text{ kJ}}{0.1415 \text{ kJ/K}} = 698 \text{ K}$$
if T > 698 K $\Rightarrow \Delta G^{\circ} < 0 \Rightarrow$ spontaneous reaction

We can say that the equation $T = \Delta H^o / \Delta S^o$ allows us to calculate the temperature at which a chemical or physical change is at equilibrium at 1 atm

atmosphere.

For example:

$$H_2O(\mathbf{l}) \longrightarrow H_2O(g)$$

 $\Delta H^o = +40.7 \text{ kJ}; \Delta S^o = +0.109 \text{ kJ/K}.$ The temperature at which ΔG^o is

zero is:

$$T = \frac{\Delta H^{\circ}}{\Delta S^{\circ}} = \frac{40.7 \text{ kJ}}{0.109 \text{ kJ/K}} = 3733 \text{ K} = 100 \,^{\circ} C$$

This is, of course, the normal boiling point of water (the temperature at which liquid water is at equilibrium with vapor at 1 atm).

§ Pressure and Concentration

It is possible to write a general relation for the free energy change, ΔG , valid under any conditions.

$$\Delta G = \Delta G^{o} + RT \ln Q$$
 (17-4)
Q: reaction quotient ----- Ch. 12
R: $8.31 \times 10^{-3} \text{ kJ/K}$

Q has the same mathematical form as the equilibrium constant, K; the difference is that the terms that appear in Q are arbitrary, initial pressures or concentrations rather than equilibrium values.

- **g** Gas enter as their partial pressures (atm) in atmosphere.
- g Species in aqueous solution enter as their molar concentrations.
- **g** Pure liquids or solids do not appear; neither does solvent in a dilute solution.

**平衡常數:

*(g): 以分壓(atm)計算
*(s)及(l)不列入計算, solvent for a solution 也不列入計算
*(aq):以 M 計算

Ex: Consider the reaction of zinc with strong acid:

$$Zn(s) + 2H^{+}(aq) \longrightarrow Zn^{2+}(aq) + H_{2}(g)$$

$$\Delta G = \Delta G^{o} + RT \ln Q$$

$$= \Delta G^{o} + RT \ln \frac{[Zn^{2+}] \cdot P_{H_{2}}}{[H^{+}]^{2}}$$

Ex. 17.7: When zinc is dissolved in a strong acid, zinc ions and hydrogen gas are produced.

$$Zn(s) + 2H^{+}(aq) \longrightarrow Zn^{2+}(aq) + H_{2}(g)$$

At 25 °C, calculate.

a). ΔG°

 $[H^+] = 3.39 \times 10^{-5}$

 $pH = -log[H^+] = -log(3.39 \times 10^{-5}) = 4.47$

- b). ΔG when $P_{H_2} = 750 \text{ mmHg}$, $Zn^{2+} = 0.10 \text{ M}$, $[H^+] = 1.0 \times 10^{-4} \text{ M}$.
- c). The pH when $\Delta G = -100.0 \text{ kJ}$, $P_{H_2} = 0.933 \text{ atm}$, $[Zn^{2+}] = 0.220 \text{ M}$ and the mass of Zn is 155g.

Ans:

a).
$$\Delta G^{o} = [1 \cdot \Delta G^{o}_{f, Zn^{2+}(aq)} + 1 \cdot \Delta G^{o}_{f, H_{2}(g)}] - [1 \cdot \Delta G^{o}_{f, Zn(s)} + 1 \cdot \Delta G^{o}_{f, H^{+}(aq)}]$$

$$= \Delta G^{o}_{f, Zn^{2+}(aq)}$$

$$= -147.1 \text{ kJ}$$
b). $\Delta G = \Delta G^{o} + \text{RTln} \frac{[Zn^{2+}] \cdot P_{H_{2}}}{[H^{+}]^{2}}$

$$= -147.1 + 8.31 \times 10^{-3} \cdot 298 \cdot \ln \frac{(0.10) \cdot (750/760)}{(1.0 \times 10^{-4})^{2}}$$

$$= -147.1 + 39.9 = -107.2 \text{ kJ}$$
c). $pH = -\log[H^{+}]$

$$\Delta G = \Delta G^{o} + \text{RTln} \frac{[Zn^{2+}] \cdot P_{H_{2}}}{[H^{+}]^{2}}$$

$$-100.0 = -147.1 + 8.31 \times 10^{-3} \cdot 298 \cdot \ln \frac{(0.22) \cdot (0.933)}{[H^{+}]^{2}}$$

$$\ln \frac{0.205}{[H^{+}]^{2}} = 19.0$$

Changes in pressure and/or concentration can have considerable effect on
$$\Delta G$$
.

Ex: The following reaction at 300 $^{\circ}$ C.

$$NH_4Cl(s) \longrightarrow NH_3(g) + HCl(g)$$
 $\Delta G^o = +13.0 \text{ kJ}$

 ΔG° "+" implies that ammonium chloride will not decompose at 300 °C to give NH₃ and HCl, both at 1 atm pressure. However, notice what

happens if
$$P_{NH_2} = P_{HCl} = 0.10$$
 atm.

Ans:

$$Q = P_{NH_3} \cdot P_{HCl} = (0.10) \cdot (0.10) = 0.0100$$

$$\Delta G = \Delta G^o + RT \ln Q$$

$$= (+13.0) + 8.31 \times 10^{-3} \cdot 573 \cdot \ln(0.0100)$$

$$= (+13.0) + (-21.9)$$

$$= -8.9 \text{ kJ}$$

This means that NH₃ and HCl can be formed, each at 0.10 atm pressure, by heating NH₄Cl to 300 $^{\circ}$ C.

Ex: A change in concentration:

$$SrCrO_4(s) \longrightarrow Sr^{2+}(aq) + CrO_4^{2-}(aq)$$
 $\Delta G^o = +25.3 \text{ kJ at } 25 \, ^oC$ $[Sr^{2+}] = 1.00 \text{ M}, [CrO_4^{2-}] = 1.00 \text{ M} \text{ at } 25 \, ^oC \Rightarrow \Delta G^o = +25.3 \text{ kJ} \Rightarrow \text{nonspontaneous}$

if
$$[Sr^{2+}] = 0.0010 \text{ M}, [CrO_4^{2-}] = 0.0010 \text{ M}$$

$$\Rightarrow \Delta G = \Delta G^{\circ} + RT \ln Q$$

$$= (+23.3) + 8.31 \times 10^{-3} \cdot 298 \cdot \ln(0.00100)^{2}$$

$$= (+23.3) + (-34.2)$$

$$= -8.9 \text{ kJ} \Rightarrow \text{spontaneous reaction}$$

if
$$[Sr^{2+}] = 0.0060 \text{ M}, [CrO_4^{2-}] = 0.0060 \text{ M}$$

$$\Rightarrow \Delta G = \Delta G^{\circ} + RT \ln Q$$

$$= (+25.3) + 8.31 \times 10^{-3} \cdot 298 \cdot \ln(0.0060)^{2}$$

$$= (+25.3) + (-25.3)$$

$$= 0 \text{ kJ} \Rightarrow \text{at equilibrium}$$

§ 17-6 The Free Energy Change and The Equilibrium

Constant

Reaction spontaneity related to:

- 1. $\Delta G^{\circ} < 0 \implies$ spontaneous.
- 2. K (equilibrium constant), $K > 1 \implies$ spontaneous.

$$\Delta G^o$$
 and K are related. $\downarrow \downarrow$ $\Delta G = \Delta G^o + \text{RTlnQ}$

When equilibrium is established Q = K and $\Delta G = 0$.

$$\Rightarrow 0 = \Delta G^{\circ} + RT \ln K$$

$$\therefore \Delta G^{\circ} = -RT \ln K$$
(17-5)

The expression for K, gases enter as their partial pressures in atmospheres and ions or molecules in solution as their molarities (M).

Ex. 17-8: Using ΔG_f^o tables in Appendix 1, calculate the solubility product constant K_{sp} for PbCl₂ at 25 $^{\circ}$ C.

Ans:

$$\begin{split} PbCl_2(s) & \longrightarrow \text{Pb}^{2+}(aq) + 2\text{Cl}^-(aq) \\ \Delta G^o &= [1 \cdot \Delta G^o_{f, \, \text{Pb}^{2+}(aq)} + 2 \cdot \Delta G^o_{f, \, \text{Cl}^-(aq)}] - (1 \cdot \Delta G^o_{f, \, \text{PbCl}_2(s)}) \\ &= [1 \cdot (-24.4) + 2 \cdot (-131.2)] - [1 \cdot (-314.1)] \\ &= +27.3 \text{ kJ} \\ \Delta G^o &= -\text{RTlnK} \\ 27.3 &= -8.31 \times 10^{-3} \cdot 298 \cdot \ln \frac{[Pb^{2+}] \cdot [\text{Cl}^-]^2}{1} \\ 27.3 &= -8.31 \times 10^{-3} \cdot 298 \cdot \ln K_{sp} \\ \ln K_{sp} &= \frac{27.3}{-2.42} = -11.0 \\ K_{sp} &= e^{-11.0} = -1.67 \times 10^{-5} \end{split}$$

The relationship $\Delta G^o = -RT \ln K$ allows us to relate the standard free energy change to the extent of reaction. Consider:





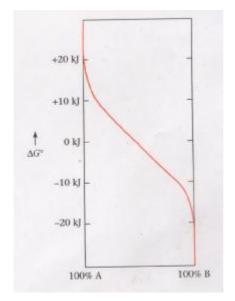


Fig. 17.8: Dependence of extent of reaction on the value of ΔG^o for the general system $A(g) \rightleftharpoons B(g)$.

 $\mathbf{g}\Delta G^{\circ} \approx 20 \text{ kJ}$, the reaction does not occur $\Rightarrow 100\% \text{ A}$

gif $\Delta G^{\circ} \leq -20$ kJ, the equilibrium constant is so large that virtually all of A converted to B $\Rightarrow 100\%$ B.

gif -20 kJ < ΔG^{o} < 20 kJ will the equilibrium mixture contain appreciable amount of both A and B.

gif $\Delta G^{o} = 0$, $K=1 \Rightarrow$ equal amount A and B.

§ 17-7 Additivity of Free Energy Change; Coupled Reaction

Free energy changes for reactions, like enthalpy or entropy, are additive. That is:

If

Reaction 3 = Reaction 1 + Reaction 2

Then

$$\Delta G_3 = \Delta G_1 + \Delta G_2$$

Metabolism (新陳代謝) reaction:

$$C_6H_{12}O_6(aq) + 6O_2(g) \longrightarrow 6CO_2(g) + 6H_2O \qquad \Delta G^o = -2870 \text{ kJ}$$

ordinarily do not serve directly as a source of energy.

Instead these reactions are use to bring about a nonspontaneous reaction:

$$ADP(aq) + HPO_4^{2-}(aq) + 2H^+(aq) \longrightarrow ATP(aq) + H_2O$$
 $\Delta G^o = +31 \text{ kJ}$ 腺苷二磷酸 腺苷三磷酸

In the coupled reaction with glucose, about 38 mole of ATP are synthesized for every one mole of glucose consumed.

$$\Delta G^{\circ} = -2870 + 38 \cdot (+31) = -1692 \text{ kJ}$$

Ex. 17.9: The lactic acid $(C_3H_6O_3(aq), \Delta G^o = -559 \text{ kJ})$ produced in muscle cells by vigorous exercise eventually is absorbed into the bloodstream, where it is metabolized back to glucose $(\Delta G_f^o = -919 \text{ kJ})$ in the liver. The reaction is:

$$2C_3H_6O_3(aq) \longrightarrow C_6H_{12}O_6(aq)$$

- a). Calculate ΔG° for this reaction, using free energies of formation.
- b). If the hydrolysis of ATP to ADP is coupled with this reaction, how many moles of ATP must react to make the process spontaneous?

Ans:

a).
$$\Delta G^o = [1 \cdot \Delta G^o_{f, C_6 H_{12} O_6(aq)}] - [2 \cdot \Delta G^o_{f, C_3 H_6 O_3(aq)}]$$

= $[1 \cdot (-919)] - [2 \cdot (-559)]$
= $+199 \text{ kJ}$

b).
$$\Delta G^{\circ} < 0$$

+199 + x \cdot (-31.0) < 0
x > 6.4 mol ATP

