## Ch 17 Spontaneity of Reaction

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The goal of this chapter is to answer a basic question: Will a given reaction occur "by itself" at a particular temperature and pressure, without exertion of any outside force?
$\Downarrow$
Is the reaction spontaneous?
$\Downarrow$
To develop a general criterion for spontaneity, we will apply the principles of thermodynamics, the science that deals with heat and energy effects.
$\Downarrow$
Three different thermodynamic functions are of value in analyzing spontaneity.

1. $\Delta H$, the change in enthalpy; a negative value of $\Delta H$ tends to make a reaction spontaneous.
2. $\Delta S$, the change in entropy; a positive value of $\Delta S$ tends to make a reaction spontaneous.
3. $\Delta G$, the change in free energy, a reaction at constant temperature and pressure will be spontaneous if $\Delta G$ is negative, no ifs, ands, or buts.

The free energy change, $\Delta G$, can be also used to:
c Determine the effect of temperature, pressure, and concentration on reaction spontaneity. (Ch. 17-5)
c Calculate the equilibrium constant for a reaction. (Ch. 17-6)
$\subseteq$ Determine whether coupled reaction will be spontaneous. (Ch. 17-7)

## § 17-1 Spontaneous Processes

Spontaneous process; for example:
$\subseteq$ An ice cube melts when added to a glass of water at room temperature.

$$
\mathrm{H}_{2} \mathrm{O}(\mathrm{~s}) \longrightarrow \mathrm{H}_{2} \mathrm{O}(\mathrm{I})
$$

$\subseteq$ A mixture of hydrogen and oxygen burns if ignited by a spark.

$$
2 \mathrm{H}_{2}(\mathrm{~g})+\mathrm{O}_{2}(\mathrm{~g}) \longrightarrow 2 \mathrm{H}_{2} \mathrm{O}(\mathrm{I})
$$

$\subseteq$ An iron (steel) tool exposed to moist air rusts.

$$
2 \mathrm{Fe}(s)+\frac{3}{2} \mathrm{O}_{2}(g)+3 \mathrm{H}_{2} \mathrm{O}(\mathrm{I}) \longrightarrow 2 \mathrm{Fe}(\mathrm{OH})_{3}(s)
$$

$\Downarrow$

These three reactions are spontaneous at $25{ }^{\circ} \mathrm{C}$ and 1 atm .
$\Downarrow$
The word "spontaneous" does not imply anything about how rapidly a reaction occurs. $\Rightarrow$ reaction rate.

If a reaction is spontaneous under a given set of conditions, the reverse reaction must be nonspontaneous.
$2 \mathrm{H}_{2} \mathrm{O}(\mathrm{I}) \longrightarrow 2 \mathrm{H}_{2}(\mathrm{~g})+\mathrm{O}_{2}(\mathrm{~g}) \quad$ nonspontaneous
$\Downarrow$
However, it is often possible to bring about a nonspontaneous reaction by supplying energy in the form of work.
$\Downarrow$
Electrolysis can be used to decompose water to elements.
$\Downarrow$
To do this, electrical energy must be furnished, perhaps from a storage battery.
$\Downarrow$
A spontaneous process is one that moves the reaction toward equilibrium. A nonspontaneous process is moves the system away from equilibrium.

## § The energy factor $\Rightarrow \Delta H$

Many spontaneous process proceed with a decrease of energy. $\Downarrow$

One might guess that spontaneous reactions would be exothermic $(\Delta H<0) . \Rightarrow$ proposed by P.M. Berthelot.
$\Downarrow$
Nearly all exothermic chemical reactions are spontaneous at 25 ${ }^{\circ} \mathrm{C}$ and 1 atm . For example:

$$
\begin{array}{ll}
2 \mathrm{H}_{2}(\mathrm{~g})+\mathrm{O}_{2}(\mathrm{~g}) \longrightarrow 2 \mathrm{H}_{2} \mathrm{O}(\mathrm{l}) & \Delta \mathrm{H}=-571.6 \mathrm{~kJ} \\
2 \mathrm{Fe}(s)+\frac{3}{2} \mathrm{O}_{2}(\mathrm{~g})+2 \mathrm{H}_{2} \mathrm{O}(\mathrm{l})+\longrightarrow 2 \mathrm{Fe}(\mathrm{OH})_{3}(s) & \Delta \mathrm{H}=-788.6 \mathrm{~kJ}
\end{array}
$$

$$
\Downarrow
$$

On the other hand, the simple rule fails for many familiar phase changes.

$$
\mathrm{H}_{2} \mathrm{O}(\mathrm{~s}) \longrightarrow \mathrm{H}_{2} \mathrm{O}(\mathrm{I}) \quad \Delta \mathrm{H}=+6.0 \mathrm{~kJ}
$$

The process is not an exothermic, but it takes place spontaneously at 1atm above $0{ }^{\circ} \mathrm{C}$.

Endothermic reactions that are nonspontaneous at room temperature often become spontaneous when the temperature is raised. For example, the decomposition of the limestone.


The endothermic reactions becomes spontaneous at high temperatures, despite that $\Delta H$ is positive.
$\Delta H$ is nearly independent of temperature (for this reaction).

## § The randomness factor $\Rightarrow \Delta S$

The direction of a spontaneous change is not always determined by the tendency for a system to go to a state of lower energy.

Nature tends to move spontaneously from a state of lower probability to one of higher probability.
$\Downarrow$
Fig. 17-2 the system has gone from a highly ordered state (all the $\mathrm{H}_{2}$ molecules on the left, all the $\mathrm{N}_{2}$ molecules on the right) to a more disordered, or random state in which the molecules are distributed evenly between two bulbs.

$$
\Downarrow
$$

In general, nature tends to move spontaneously from more ordered to
more random states.

## § 17-2 Entropy, S

Entropy is often described as a measure of disorder or randomness. $\Downarrow$

## Boltzmann Relationships:

The absolute entropy $S$ of the system is proportional to the natural logarithm of the possible combinations.

$$
S=\mathrm{k} \ln \Omega
$$

$\Omega$ : the number of possible combinations.
k: proportional constant, "Boltzmann constant"

$$
\mathrm{k}=R / N_{a} ; 1.381 \times 10^{-23} \mathrm{~J} / \mathrm{K}
$$

$\Downarrow$
An increase in the number of possible microstates (i.e., disorder) results in an increase of entropy.
$\Downarrow$
In general, the more random the state, the larger the number of its possible microstates, the more probable the state, thus the greater its entropy.

Entropy is a state property. That is, the entropy depends only on the state of a system, not on its history. The entropy change is determined by the entropies of the final and the initial states, not on the path followed from one state to another.

$$
\Delta S=\mathrm{S}_{\text {final }}-\mathrm{S}_{\text {initial }}
$$



Fig. 17-4: Molar entropy of ammonia as a function of temperature.

Several factors influence the amount of entropy:
g A liquid has a higher entropy than the solid from which it is formed. $S(\mathrm{I})>\mathrm{S}(\mathrm{s})$
$S(\mathrm{I})$ : molecules can occupy many more positions as they move away from the lattice.
$\mathrm{S}(\mathrm{s})$ : molecules are confined to fixed position.
$g$ A gas has a higher entropy than the liquid from which it is formed. $S(g)>\mathrm{S}(\mathrm{I})$
$g$ Increasing the temperature of a substance increase its entropy.
$T \uparrow \Rightarrow \mathrm{~S} \uparrow$

The third law of thermodynamics:
Absolute entropy approaches zero as the absolute temperature approaches zero.
gMelting and vaporization are accompanied by relatively large increase in entropy.

Ex. 17.1: Predict whether $\Delta S$ is positive or negative for each of the following processes:
a). taking dry ice from a freezer where its temperature is $-80^{\circ} \mathrm{C}$ and allowing it to warm to room temperature.
b). dissolving bromine in hexane.
c). condensing gaseous bromine to liquid bromine.

Ans:
a). $T \uparrow \Rightarrow \mathrm{~S} \uparrow \quad \therefore \Delta \mathrm{~S} \uparrow$
b). Q more random $\mathrm{S} \uparrow \Rightarrow \Delta \mathrm{S}>0$
c). $S(g)>\mathrm{S}(\mathrm{I}) \therefore \mathrm{g} \rightarrow \mathrm{I} \quad \Delta \mathrm{S} \downarrow \Rightarrow \Delta \mathrm{S}<0$

## § Standard molar entropies, $S^{o}$

The entropy of a substance, unlike its enthalpy, can be evaluated directly.

$$
\Downarrow
$$

The only state property which has the absolute value.
g elements have nonzero standard entropies.
g standard molar entropies of pure substance (elements and compounds) are always positive quantities. $\left(S^{\circ}>0\right)$
g aqueous ions may have negative values. Ex: $\mathrm{S}_{F(\text { (aq) }}^{o}=-13.8 \mathrm{~J} / \mathrm{mol} \cdot \mathrm{K}$
Defined: $\quad S_{H^{+}(a q)}^{o}=0$

Among substances of similar structure and physical state, entropy usually increases with molar mass.

|  | $\mathrm{CH}_{4}(\mathrm{~g})$ | $\mathrm{S}^{o}=186.2 \mathrm{~J} / \mathrm{mol} \cdot \mathrm{K}$ |
| :--- | :--- | :--- |
| $\mathrm{Ex}:$ | $\mathrm{C}_{2} H_{6}(g)$ | $\mathrm{S}^{o}=229.5 \mathrm{~J} / \mathrm{mol} \cdot \mathrm{K}$ |
|  | $\mathrm{C}_{3} H_{8}(\mathrm{~g})$ | $\mathrm{S}^{o}=269.9 \mathrm{~J} / \mathrm{mol} \cdot \mathrm{K}$ |

## $\S \Delta S^{o}$ for reactions

$\Delta S^{\circ}$, the standard entropy change, for reactions can be determined as:

$$
\begin{equation*}
\Delta S^{o}=\sum S_{\text {products } s}^{o}-\sum S_{\text {reactants }}^{o} \tag{17-1}
\end{equation*}
$$

Ex: Consider the reaction, calculate $\Delta S^{\circ}$ :

$$
\begin{aligned}
& \mathrm{CaCO}_{3}(\mathrm{~s}) \longrightarrow \mathrm{CaO}(\mathrm{~s})+\mathrm{CO}_{2}(g) \\
& \begin{aligned}
& \Delta S^{o}=\sum S_{\text {products }}^{o}-\sum S_{\text {reactants }}^{o} \\
& \quad= {\left[1 \cdot \mathrm{~S}_{\mathrm{CaO}(S)}^{o}+1 \cdot \mathrm{~S}_{\mathrm{CO}_{2}(g)}^{o}\right]-\left(1 \cdot \mathrm{~S}_{C a C O_{3}(S)}^{o}\right) } \\
& \quad= {[1 \cdot 39.8+1 \cdot 213.6]-(1 \cdot 92.9) } \\
&=+160.5 \mathrm{~J} / \mathrm{K} \\
& \mathrm{Q} \quad \Delta \mathrm{n}_{g}=1-0=1 \quad \therefore \Delta S^{o}>0 \\
& \Downarrow
\end{aligned}
\end{aligned}
$$

A reaction that results in an increase in the number of moles of gas is accompanied by an increase in entropy. Conversely, if the number of moles of gas decreases, $\Delta S^{\circ}$ is a negative quantity.

$$
\begin{gathered}
\Delta \mathrm{n}_{g} "+" \Rightarrow \Delta S^{o}>0 \\
\Delta \mathrm{n}_{g} "-" \Rightarrow \Delta S^{o}<0
\end{gathered}
$$

Ex.

$$
\begin{aligned}
& 2 \mathrm{H}_{2}(g)+\mathrm{O}_{2}(g) \longrightarrow 2 \mathrm{H}_{2} \mathrm{O}(\mathrm{I}) \\
& \begin{aligned}
\Delta S^{o} & =\sum S_{\text {products }}^{o}-\sum S_{\text {reactants }}^{o} \\
& =\left(2 \cdot \mathrm{~S}_{H_{2} O(1)}^{o}\right)-\left[2 \cdot \mathrm{~S}_{H_{2}(g)}^{o}+1 \cdot \mathrm{~S}_{2_{2}(g)}^{o}\right] \\
& =(2 \cdot 69.9)-[2 \cdot(130.6)+1 \cdot(205.0)] \\
& =-326.4 \mathrm{~J} / \mathrm{K} \\
\Delta \mathrm{n}_{g} & =0-(2+1)=-3 \quad "-" \Rightarrow \Delta S^{o}<0
\end{aligned}
\end{aligned}
$$

Ex. 17.2: Calculate $\Delta S^{o}$ for
a). dissolving one mole of calcium hydroxide in water.
b). the combustion of one gram of methane to form carbon dioxide and liquid water.
Ans:
a). $\quad \mathrm{Ca}(\mathrm{OH})_{2}(s) \longrightarrow \mathrm{Ca}^{2+}(a q)+2 \mathrm{OH}^{-}(a q)$ $\Delta S^{o}:+83.4 \quad-53.1 \quad-10.8 \mathrm{~J} / \mathrm{K}$

$$
\Delta S^{o}=\left[1 \cdot \mathrm{~S}_{\mathrm{Ca}^{2+}(a q)}^{o}+2 \cdot \mathrm{~S}_{O H^{-}(a q)}^{o}\right]-\left(1 \cdot \mathrm{~S}_{\mathrm{Ca(OH)}_{2}(a q)}^{o}\right)
$$

$$
=[1 \cdot(-53.1)+2 \cdot(-10.8)]-(1 \cdot 83.40)
$$

$$
=-158.1 \mathrm{~J} / \mathrm{K}<0 \quad \therefore \text { nonspontaneous }
$$

b).

$$
\begin{aligned}
& \mathrm{CH}_{4}(g)+2 \mathrm{O}_{2}(g) \longrightarrow \mathrm{CO}_{2}(g)+2 \mathrm{H}_{2} \mathrm{O}(\mathrm{l}) \\
\Delta S^{o}: & +186.2+205.0 \quad+213.6+69.9 \\
\Delta S^{o}= & {\left[1 \cdot \mathrm{~S}_{C O_{2}(g)}^{o}+2 \cdot \mathrm{~S}_{H_{2} O(\mathrm{l})}^{o}\right]-\left[1 \cdot \mathrm{~S}_{C H_{4}(g)}^{o}+2 \mathrm{~S}_{O_{2}(g)}^{o}\right] } \\
= & {[1 \cdot 213.6+2 \cdot 69.9]-[1 \cdot 186.2+2 \cdot 205.0] } \\
= & -242.8 \mathrm{~J} / \mathrm{K} \text { for the combustion of one mole of } \mathrm{CH}_{4}(\mathrm{~g})
\end{aligned}
$$

For the combustion of one gram of $\mathrm{CH}_{4}(g)$ is:

$$
\frac{1.00}{16.04} \cdot(-242.8)=-15.14 \mathrm{~J} / \mathrm{K}
$$

## § The second law of thermodynamics:

The relationship between entropy and spontaneity can be expressed through a basic principle of nature known as the second law of thermodynamics.
$\Downarrow$
For an isolated system, if a spontaneous change occurs, it occurs with a concurrent increase in the entropy of the system.

$$
\Delta S_{\text {universe }}=\left(\Delta \mathrm{S}_{\text {system }}+\Delta \mathrm{S}_{\text {surroundings }}\right)>0 \quad \text { spontaneous system }
$$

For many spontaneous processes, the entropy change for the system is negative quantity. Consider, for example, the rusting of iron, a spontaneous process:

$$
\begin{aligned}
& 2 \mathrm{Fe}(\mathrm{~s})+\frac{3}{2} \mathrm{O}_{2}(g)+3 \mathrm{H}_{2} O(\mathrm{l}) \longrightarrow 2 \mathrm{Fe}(\mathrm{OH})_{3}(s) \\
& \Delta S^{o}: 27.3 \quad 205.0 \quad 69.9 \quad 106.7 \\
& \Delta S^{o}=(2 \cdot 106.7)-\left[2 \cdot(27.3)+\frac{3}{2} \cdot(205.0)+3 \cdot(69.9)\right] \\
&=-358.4 \mathrm{~J} / \mathrm{K}<0 \quad \therefore \text { nonspontaneous } \\
& \Downarrow
\end{aligned}
$$

All the law requires is that the entropy change of the surroundings be greater than $358.4 \mathrm{~J} / \mathrm{K}$, so that $\Delta S_{\text {unierese }}>0$.
$\Downarrow$

$$
\begin{aligned}
& \Delta S_{\text {unineress }}=\left(\Delta S_{\text {sssem }}+\Delta \mathrm{S}_{\text {surroumdings }}\right)>0 \\
&=(-358.4)+\Delta \mathrm{S}_{\text {survoundings }}>0 \\
& \Rightarrow \Delta \mathrm{~S}_{\text {surroundings }}>358.4 \mathrm{~J} / \mathrm{K}
\end{aligned}
$$

$\Downarrow$
The calculate the entropy change for the surroundings is not easy. $\Downarrow$
We deals only with the thermodynamic properties of chemical systems.

## § 17-3 Free Energy, G

To put the enthalpy and the entropy together in such a way as to arrive at a single function whose sign will determine whether a reaction is spontaneous.
$\Downarrow$
J. W. Gibbs introduced a new quantity.
$\Downarrow$
Gibbs free energy, G. $\mathrm{G}=\mathrm{H}-\mathrm{TS}$
$\Downarrow$
For a reaction taking place at constant pressure and temperature, $\Delta G$ represents that portion of the total energy change that is available (i.e., "free") to do useful work. For example, $\Delta G$ for a reaction is -270 kJ , it is possible to obtain 270 kJ of useful work from the reaction. Conversely, if $\Delta G$ is +270 kJ , at least that much energy in the form of work must be supplied to
make the reaction take place.

G is a state property.
$\Downarrow$
$\Delta G$ for a reaction depends only on the nature of products and reactants and the conditions (temperature, pressure, and concentration).
c If $\Delta G$ is negative, the reaction is spontaneous.
$\subseteq$ If $\Delta G$ is positive, the reaction will not take place spontaneously. Instead, the reverse reaction will be spontaneous.
$\subseteq$ If $\Delta G$ is 0 , the system is at equilibrium; there is no tendency for reaction to occur in either direction.
$\Delta G$ is a measure of the "driving force of a reaction". Reactions, at constant pressure and temperature, go in such a direction as to decrease the free energy of the system.


Fig. 17.5. Sign of free energy and spontaneity.
$\subseteq$ If the products at the specific conditions of temperature, pressure, and concentration have a lower free energy than the reactants ( $\mathrm{G}_{\text {products }}<$ $\left.\mathrm{G}_{\text {reactants }} \Rightarrow \Delta \mathrm{G}<0\right)$, the forward reaction will occur.
$\subseteq$ If the reverse is true $\left(\left(\mathrm{G}_{\text {reactants }}<\mathrm{G}_{\text {products }} \Rightarrow \Delta \mathrm{G}>0\right)\right.$, the reverse reaction is spontaneous.
$\subseteq$ If $\mathrm{G}_{\text {products }}=\mathrm{G}_{\text {reactants }}(\Rightarrow \Delta \mathrm{G}=0)$ there is no driving force to make the reaction go in either direction.

## § Relation among $\Delta G, \Delta H$, and $\Delta S$

Gibbs-Helmholtz equation: $\Delta G=\Delta \mathrm{H}-\mathrm{T} \Delta \mathrm{S}$ at constant temperature.

Two factors tend to make $\Delta G$ negative and hence lead to a spontaneous reaction:

1. A negative value of $\Delta H$ :

Exothermic reaction $(\Delta H<0)$ tend to be spontaneous. On the molecule level, this means that there will be a tendency to form "strong" bonds at the expense of "weak" ones.
2. A positive value of $\Delta S$ :
$\Delta S>0 \Rightarrow-\mathrm{T} \Delta S<0$. Hence there will be a tendency for a reaction to be spontaneous if the products are less ordered than the reactants.

## § 17-4 Standard Free Energy Change, $\Delta G^{\circ}$

Standard conditions: $\mathrm{T}=25^{\circ} \mathrm{C}$
Gas: partial pressure $=1 \mathrm{~atm}$.
Molar concentration of ions or molecules in solution $=1 \mathrm{M}$.

$$
\Delta G^{o}=\Delta \mathrm{H}^{o}-\mathrm{T} \Delta \mathrm{~S}^{o}
$$

1. If $\Delta G^{o}$ is negative, the reaction is spontaneous at standard conditions.

$$
\mathrm{CaO}(s)+\mathrm{CO}_{2}(g, 1 \mathrm{~atm}) \longrightarrow \mathrm{CaCO}_{3}(s) \quad \Delta \mathrm{G}^{o}=-130.4 \mathrm{~kJ}
$$

2. If $\Delta G^{o}$ is positive, the reaction is nonspontaneous at standard conditions.

$$
\operatorname{AgCl}(s) \longrightarrow \mathrm{Ag}^{+}(a q, 1 \mathrm{M})+\mathrm{Cl}^{-}(a q, 1 \mathrm{M}) \quad \Delta \mathrm{G}^{o}=+55.7 \mathrm{~kJ}
$$

The reverse reaction is spontaneous.
3. If $\Delta G^{o}$ is 0 , the system is at equilibrium at standard conditions.

$$
\mathrm{H}_{2} \mathrm{O}(\mathrm{l}) \longrightarrow \mathrm{H}_{2} \mathrm{O}(\mathrm{~g}, 1 \mathrm{~atm}) \quad \Delta \mathrm{G}^{o}=0 \mathrm{~kJ}
$$

## § Calculation of $\Delta G^{o}$ at $25{ }^{\circ} \mathrm{C}$; free energy of formation

Ex. 17.3: Calcium sulfate, $\mathrm{CaSO}_{4}$, is used as a drying agent and sold under trade name Drierite. For the reaction:

$$
\mathrm{CaSO}_{4}(s) \longrightarrow \mathrm{Ca}^{2+}(a q)+\mathrm{SO}_{4}{ }^{2-}(a q)
$$

Calculate $a) . \Delta H^{o}$ b). $\Delta S^{o}$ c). $\Delta G^{o}$ at $25^{\circ} C$.
Ans:

$$
\begin{array}{rccc}
\mathrm{CaSO}_{4}(s) \longrightarrow \mathrm{Ca}^{2+}(a q)+\mathrm{SO}_{4}{ }^{2-}(a q) \\
\Delta H_{f}^{o}: & -1434.1 & -542.8 & -909.3 \mathrm{~kJ} \\
S^{o}: & -106.7 & -53.1 & +20.1
\end{array}
$$

a). $\Delta H^{o}=\sum \Delta H_{f, \text { products }}^{o}-\sum \Delta H_{f, \text { reactants }}^{o}$

$$
=\left[1 \cdot \Delta \mathrm{H}_{f, C a^{2+}(a q)}^{o}+1 \cdot \Delta \mathrm{H}_{f, S O_{4}^{2-(a q)}}^{o}\right]-\left(1 \cdot \Delta \mathrm{H}_{f, \mathrm{CaSO}_{4}(s)}^{o}\right)
$$

$$
=[1 \cdot(-542.8)+1 \cdot(-909.3)]-[1 \cdot(-1434.1)]
$$

$$
=-18.0 \mathrm{~kJ}
$$

b). $\Delta S^{o}=\sum S_{\text {products }}^{o}-\sum S_{\text {reactants }}^{o}$

$$
=[1 \cdot(-53.1)+1 \cdot(+20.1)]-[1 \cdot(-106.7)]
$$

$$
=-139.7 \mathrm{~J} / \mathrm{K}=-0.1397 \mathrm{~kJ} / \mathrm{K}
$$

c). $\Delta G^{o}=\Delta \mathrm{H}^{o}-\mathrm{T} \Delta \mathrm{S}^{o}$

$$
\begin{aligned}
& =(-18.0)-298 \cdot(-0.1397) \\
& =+23.6 \mathrm{~kJ}
\end{aligned}
$$

## § Standard free energy of formation, $\Delta G_{f}^{o}$

$\Delta G_{f}^{o}$ is analogous to the enthalpy of formation, $\Delta H_{f}^{o}$.

It is defined as the free energy per mole when a compound is formed from the elements in their stable states at 1 atm.

For most compounds, $\Delta G_{f}^{o}$ is a negative quantity, which means that the compound can be formed spontaneously from the elements. This is true for water:

$$
\mathrm{H}_{2}(g)+\frac{1}{2} \mathrm{O}_{2}(g) \longrightarrow \mathrm{H}_{2} O(\mathrm{I}) \quad \Delta \mathrm{G}_{f}^{o}=-237.2 \mathrm{~kJ} / \mathrm{mol}
$$

A few compounds, including acetylene, have positive free energies of formation $\left(\Delta G_{f, C_{2} H_{2}(\mathrm{I})}^{o}=+209.2 \mathrm{~kJ} / \mathrm{mol}\right)$. These compounds cannot be made from the elements at ordinary temperatures and pressures; they are potentially unstable with respect to the elements. In the case of acetylene, the reaction:

$$
C_{2} H_{2}(g) \longrightarrow 2 \mathrm{C}(\mathrm{~s})+\mathrm{H}_{2}(g) \quad \Delta \mathrm{G}^{o}=-209.2 \mathrm{~kJ}
$$

occurs with explosive violent unless special precaution are taken.

$$
\begin{equation*}
\Delta G_{\text {reaction }}^{o}=\sum \Delta G_{f, p \text { producs }}^{o}-\sum \Delta G_{f, \text {,eactants }}^{o} \tag{17-3}
\end{equation*}
$$

$\Delta G_{\text {reacion }}^{o}$ is valid only at the temperature at which $\Delta G_{f}^{o}$ data are tabulated, in this case $25^{\circ} \mathrm{C} . \Delta G^{\circ}$ varies considerably with temperature, so this approach is not even approximately valid at other temperature.

Ex. 17.4: Using $\Delta G_{f}^{o}$ values from Appendix 1, calculate the standard free energy change at $25^{\circ} \mathrm{C}$ for the reaction referred to in Example 17.3. Ans:

$$
\begin{aligned}
& \mathrm{CaSO}_{4}(s) \longrightarrow \\
\Delta G_{f}^{o}: & -1321.8 \quad-5 \mathrm{Ca}^{2+}(a q)+\mathrm{SO}_{4}^{2-}(a q) \\
\Delta G^{o}= & \sum \Delta G_{f, \text { products }}^{o}-\sum \Delta G_{f, \text { reactants }}^{o} \\
= & {[1 \cdot \Delta 44.5 \mathrm{~kJ}} \\
= & {[1 \cdot(-553.6)+1 \cdot(-774.5)]-[1 \cdot(-1321.8)] } \\
= & +23.7 \mathrm{~kJ} \approx \text { Ex. } 17.3:+23.6 \mathrm{~kJ}
\end{aligned}
$$

## § Calaulation of $\Delta G^{o}$ at other temperature, $\Delta G_{T}$

To a good degree of approximation, the temperature variation of $\Delta H^{\circ}$ and $\Delta S^{\circ}$ can be neglected. $\because \Delta H$ and $\Delta S$ always change in the same direction as the temperature changes.

$$
\Delta G^{o}=\Delta \mathrm{H}^{o}-\mathrm{T} \Delta \mathrm{~S}^{o}
$$

At temperatures other than $25^{\circ} \mathrm{C}$, you need only change the value of T .
Ex. 17.5: Iron, a large component of steel, is obtained by reducing iron (III) oxide (present in hematite ore) with hydrogen in a blast furnace. Steam is a byproduct of the reaction. Calculate $\Delta G^{\circ}$ at $230^{\circ} \mathrm{C}$ for the reduction of one mole of $\mathrm{Fe}_{2} \mathrm{O}_{3}$.
Ans:

| $\mathrm{Fe}_{2} \mathrm{O}_{3}(s)+3 \mathrm{H}_{2}(g) \longrightarrow$ |  |  |  | $2 \mathrm{Fe}(\mathrm{s})+3 \mathrm{H}_{2} \mathrm{O}(\mathrm{g})$ |
| :--- | :---: | :---: | :---: | :---: |
| $\Delta H_{f}^{o}:$ | -824.2 | 0 | 0 | -241.8 kJ |
| $S^{o}:$ | 87.4 | 130.6 | 27.3 | $188.7 \mathrm{~J} / \mathrm{K}$ |

$$
\begin{aligned}
\Delta H^{o} & =\sum \Delta H_{f, \text { products }}^{o}-\sum \Delta H_{f, r \text { reactants }}^{o} \\
& =\left(3 \cdot \Delta \mathrm{H}_{f, H_{2} o(1)}^{o}\right)-\left(1 \cdot \Delta \mathrm{H}_{f, F e_{2} O_{3}(s)}^{o}\right) \\
& =[3 \cdot(-241.8)]-[1 \cdot(-824.2)] \\
& =+98.8 \mathrm{~kJ} \\
\Delta S^{o}= & \sum S_{\text {products }}^{o}-\sum S_{\text {reactants }}^{o} \\
& =[2 \cdot(27.3)+3 \cdot(188.7)]-[1 \cdot(87.4)+3 \cdot(130.6)] \\
& =+141.5 \mathrm{~J} / \mathrm{K}=+0.1415 \mathrm{~kJ} / \mathrm{K} \\
\Delta G^{o}= & \Delta \mathrm{H}^{o}-\mathrm{T} \Delta \mathrm{~S}^{o} \\
& =(+98.8)-503 \cdot(+0.1415) \\
& =+27.6 \mathrm{~kJ}
\end{aligned}
$$

$\Delta G^{o}$, unlike $\Delta H^{o}$ and $\Delta \mathrm{S}^{o}$, is strongly dependent on temperature. This comes about, of course, because of the T is in the Gibbs-Helmholtz equation:

$$
\Delta G^{o}=\Delta \mathrm{H}^{o}-\mathrm{T} \Delta \mathrm{~S}^{o}
$$



Fig. 17.6: Variation of $\Delta G^{o}$ with T.

A plot of $\Delta G^{o}$ versus T, should be linear, with a slope of $-\Delta S^{o}$ and a y-intercept (at 0 K ) of $\Delta H^{o}$.

## § 17-5 Effect of Temperature, Pressure and <br> Concentration on Reaction Spontaneity

A change in reaction conditions can, and often does, change the direction in which a reaction occurs spontaneously.

$$
\Delta G^{o}=\Delta \mathrm{H}^{o}-\mathrm{T} \Delta \mathrm{~S}^{o}
$$

is readily applied to deduce the effect of temperature on reaction spontaneity.
§ Temperature
Table 17-2: Effect of temperature on reaction spontaneity.

| Case | $\Delta H^{\circ}$ | $\Delta S^{\circ}$ | $\Delta \mathrm{G}^{o}=\Delta \mathrm{H}^{o}-\mathrm{T} \Delta \mathrm{S}^{o}$ | Remarks |
| :---: | :---: | :---: | :---: | :---: |
| I | - | + | always - | Spontaneous at all T; reverse reaction always nonspontaneous. |
| II | + | - | always + | Nonspontaneous at all T. |
| III | + | + | + at low T <br> - at high T | Nonspontaneous at low T, becomes spontaneous at high T. |
| IV | - | - | - at low T <br> + at high T | Spontaneous at low T, becomes nonspontaneous as T is raised. |

If $\Delta H^{\circ}$ and $\Delta S^{\circ}$ have opposite signs (case I and II), it is impossible to reverse the direction for spontaneity by a change in temperature alone.

Case III and IV, $\Delta H^{\circ}$ and $\Delta S^{\circ}$ have the same sign, at low temperatures, $\Delta H^{\circ}$ predominates, and the exothermic reaction, which may be either the forward or the reverse reaction, occurs. As the temperature raises, the quantity $T \Delta S^{\circ}$ increase in magnitude and eventually exceeds $\Delta H^{\circ}$. At high temperatures, the reaction that leads to an increase in entropy occurs.

In most case, $25^{\circ} \mathrm{C}$ is a "low" temperature, at least at a pressure of 1 atm . This explains why exothermic reactions are usually spontaneous at room temperature and atmospheric pressure.

Ex. 17.6: At what temperature does $\Delta G^{o}$ becomes zero for the reaction consider in Ex.17.5.

$$
\mathrm{Fe}_{2} O_{3}(s)+3 \mathrm{H}_{2}(g) \longrightarrow 2 \mathrm{Fe}(\mathrm{~s})+3 \mathrm{H}_{2} \mathrm{O}(\mathrm{l})
$$

Ans:

$$
\begin{aligned}
\Delta G^{o} & =\Delta \mathrm{H}^{o}-\mathrm{T} \Delta \mathrm{~S}^{o}=0 \\
\mathrm{~T} & =\frac{\Delta \mathrm{H}^{o}}{\Delta \mathrm{~S}^{o}}=\frac{98.8 \mathrm{~kJ}}{0.1415 \mathrm{~kJ} / \mathrm{K}}=698 \mathrm{~K} \\
\text { if } \mathrm{T} & >698 \mathrm{~K} \Rightarrow \Delta G^{o}<0 \Rightarrow \text { spontaneous reaction }
\end{aligned}
$$

We can say that the equation $T=\Delta H^{\circ} / \Delta S^{\circ}$ allows us to calculate the temperature at which a chemical or physical change is at equilibrium at 1 atm
atmosphere．

For example：

$$
\mathrm{H}_{2} \mathrm{O}(\mathrm{I}) \longrightarrow \mathrm{H}_{2} \mathrm{O}(\mathrm{~g})
$$

$\Delta H^{o}=+40.7 \mathrm{~kJ} ; \Delta S^{o}=+0.109 \mathrm{~kJ} / \mathrm{K}$ ．The temperature at which $\Delta G^{o}$ is
zero is：
$T=\frac{\Delta \mathrm{H}^{o}}{\Delta \mathrm{~S}^{o}}=\frac{40.7 \mathrm{~kJ}}{0.109 \mathrm{~kJ} / \mathrm{K}}=3733 \mathrm{~K}=100^{\circ} \mathrm{C}$
This is，of course，the normal boiling point of water（the temperature at which liquid water is at equilibrium with vapor at 1 atm）．

## § Pressure and Concentration

It is possible to write a general relation for the free energy change，$\Delta G$ ， valid under any conditions．

$$
\begin{equation*}
\Delta G=\Delta G^{o}+\mathrm{RT} \ln \mathrm{Q} \tag{17-4}
\end{equation*}
$$

Q：reaction quotient－－－－－－Ch． 12
R： $8.31 \times 10^{-3} \mathrm{~kJ} / \mathrm{K}$

Q has the same mathematical form as the equilibrium constant， K ；the difference is that the terms that appear in Q are arbitrary，initial pressures or concentrations rather than equilibrium values．
c Gas enter as their partial pressures（atm）in atmosphere．
c Species in aqueous solution enter as their molar concentrations．
$\subseteq$ Pure liquids or solids do not appear；neither does solvent in a dilute solution．
＊＊平衡常數：
＊（g）：以分壓（atm）計算
＊（s）及（l）不列入計算，solvent for a solution 也不列入計算 ＊（aq）：以 M 計算

Ex：Consider the reaction of zinc with strong acid：

$$
\mathrm{Zn}(s)+2 \mathrm{H}^{+}(a q) \longrightarrow \mathrm{Zn}^{2+}(a q)+\mathrm{H}_{2}(g)
$$

$$
\Delta G=\Delta G^{o}+\mathrm{RT} \ln \mathrm{Q}
$$

$$
=\Delta G^{o}+\mathrm{RT} \ln \frac{\left[Z n^{2+}\right] \cdot \mathrm{P}_{H_{2}}}{\left[H^{+}\right]^{2}}
$$

Ex. 17.7: When zinc is dissolved in a strong acid, zinc ions and hydrogen gas are produced.

$$
\mathrm{Zn}(s)+2 \mathrm{H}^{+}(a q) \longrightarrow \mathrm{Zn}^{2+}(a q)+\mathrm{H}_{2}(g)
$$

At $25{ }^{\circ} \mathrm{C}$, calculate.
a). $\Delta G^{o}$
b). $\Delta G$ when $P_{H_{2}}=750 \mathrm{mmHg}, \mathrm{Zn}^{2+}=0.10 \mathrm{M},\left[\mathrm{H}^{+}\right]=1.0 \times 10^{-4} \mathrm{M}$.
c). The pH when $\Delta G=-100.0 \mathrm{~kJ}, \mathrm{P}_{\mathrm{H}_{2}}=0.933 \mathrm{~atm},\left[\mathrm{Zn}^{2+}\right]=0.220 \mathrm{M}$ and the mass of Zn is 155 g .
Ans:
a). $\Delta \mathrm{G}^{o}=\left[1 \cdot \Delta \mathrm{G}_{f, \mathrm{Zn}^{2+}(a q)}^{o}+1 \cdot \Delta \mathrm{G}_{f, \mathrm{H}_{2}(g)}^{o}\right]-\left[1 \cdot \Delta \mathrm{G}_{f, \mathrm{Zn}(s)}^{o}+1 \cdot \Delta \mathrm{G}_{f, \mathrm{H}^{+}(a q)}^{o}\right]$

$$
=\Delta \mathrm{G}_{f, Z n^{2+}(a q)}^{o}
$$

$$
=-147.1 \mathrm{~kJ}
$$

b). $\Delta \mathrm{G}=\Delta G^{o}+\mathrm{R} \ln \frac{\left[Z n^{2+}\right] \cdot \mathrm{P}_{H_{2}}}{\left[H^{+}\right]^{2}}$

$$
\begin{aligned}
& =-147.1+8.31 \times 10^{-3} \cdot 298 \cdot \ln \frac{(0.10) \cdot(750 / 760)}{\left(1.0 \times 10^{-4}\right)^{2}} \\
& =-147.1+39.9=-107.2 \mathrm{~kJ}
\end{aligned}
$$

c). $\mathrm{pH}=-\log \left[\mathrm{H}^{+}\right]$

$$
\begin{aligned}
\Delta \mathrm{G} & =\Delta G^{o}+\mathrm{RT} \ln \frac{\left[Z n^{2+}\right] \cdot \mathrm{P}_{H_{2}}}{\left[H^{+}\right]^{2}} \\
-100.0 & =-147.1+8.31 \times 10^{-3} \cdot 298 \cdot \ln \frac{(0.22) \cdot(0.933)}{\left[H^{+}\right]^{2}}
\end{aligned}
$$

$$
\ln \frac{0.205}{\left[H^{+}\right]^{2}}=19.0
$$

$$
\left[H^{+}\right]=3.39 \times 10^{-5}
$$

$$
\mathrm{pH}=-\log \left[\mathrm{H}^{+}\right]=-\log \left(3.39 \times 10^{-5}\right)=4.47
$$

Changes in pressure and/or concentration can have considerable effect on $\Delta G$.

Ex: The following reaction at $300{ }^{\circ} \mathrm{C}$.

$$
\mathrm{NH}_{4} \mathrm{Cl}(\mathrm{~s}) \longrightarrow \mathrm{NH}_{3}(\mathrm{~g})+\mathrm{HCl}(\mathrm{~g}) \quad \Delta \mathrm{G}^{o}=+13.0 \mathrm{~kJ}
$$

$\Delta G^{o}$ " + " implies that ammonium chloride will not decompose at $300^{\circ} \mathrm{C}$ to give $\mathrm{NH}_{3}$ and HCl , both at 1 atm pressure. However, notice what happens if $P_{\mathrm{NH}_{3}}=\mathrm{P}_{\text {HCl }}=0.10 \mathrm{~atm}$.

Ans:

$$
\begin{aligned}
Q & =\mathrm{P}_{N H_{3}} \cdot \mathrm{P}_{H C l}=(0.10) \cdot(0.10)=0.0100 \\
\Delta G & =\Delta G^{o}+\mathrm{RTln} \mathrm{Q} \\
& =(+13.0)+8.31 \times 10^{-3} \cdot 573 \cdot \ln (0.0100) \\
& =(+13.0)+(-21.9) \\
& =-8.9 \mathrm{~kJ}
\end{aligned}
$$

This means that $\mathrm{NH}_{3}$ and HCl can be formed, each at 0.10 atm pressure, by heating $\mathrm{NH}_{4} \mathrm{Cl}$ to $300{ }^{\circ} \mathrm{C}$.

Ex: A change in concentration:

$$
\begin{aligned}
& \begin{aligned}
\mathrm{SrCrO}_{4}(s) & \longrightarrow \mathrm{Sr}^{2+}(a q)+\mathrm{CrO}_{4}{ }^{2-}(a q) \quad \Delta \mathrm{G}^{o}=+25.3 \mathrm{~kJ} \text { at } 25^{\circ} \mathrm{C} \\
{\left[\mathrm{Sr}^{2+}\right]=} & 1.00 \mathrm{M},\left[\mathrm{CrO}_{4}{ }^{2-}\right]=1.00 \mathrm{M} \text { at } 25^{\circ} \mathrm{C} \Rightarrow \Delta \mathrm{G}^{o}=+25.3 \mathrm{~kJ} \Rightarrow \text { nonspontaneous } \\
\text { if } \quad\left[\mathrm{Sr}^{2+}\right] & =0.0010 \mathrm{M},\left[\mathrm{CrO}_{4}{ }^{2-}\right]=0.0010 \mathrm{M} \\
\Rightarrow \Delta G & =\Delta G^{o}+\mathrm{RT} \operatorname{lnQ} \\
& =(+23.3)+8.31 \times 10^{-3} \cdot 298 \cdot \ln (0.00100)^{2} \\
& =(+23.3)+(-34.2) \\
& =-8.9 \mathrm{~kJ} \Rightarrow \text { spontaneous reaction }
\end{aligned} \\
& \text { if } \begin{aligned}
{\left[\mathrm{Sr}^{2+}\right] } & =0.0060 \mathrm{M},\left[\mathrm{CrO}_{4}^{2-}\right]=0.0060 \mathrm{M} \\
\Rightarrow \Delta G & =\Delta G^{o}+\mathrm{RTlnQ} \\
& =(+25.3)+8.31 \times 10^{-3} \cdot 298 \cdot \ln (0.0060)^{2} \\
& =(+25.3)+(-25.3) \\
& =0 \mathrm{~kJ} \Rightarrow \text { at equilibrium }
\end{aligned}
\end{aligned}
$$

## § 17-6 The Free Energy Change and The Equilibrium Constant

Reaction spontaneity related to:

1. $\Delta G^{o}<0 \Rightarrow$ spontaneous.
2. K (equilibrium constant), $\mathrm{K}>1 \Rightarrow$ spontaneous.
$\Downarrow$
$\Delta G^{o}$ and $\mathrm{K} \quad$ are related.
$\Downarrow$
$\Delta G=\Delta G^{o}+\mathrm{RTln} \mathrm{Q}$

When equilibrium is established $\mathrm{Q}=\mathrm{K}$ and $\Delta G=0$.

$$
\begin{align*}
\Rightarrow & 0 & =\Delta G^{o}+\mathrm{RT} \ln \mathrm{~K} \\
\therefore & \Delta G^{o} & =-\mathrm{RT} \ln \mathrm{~K} \tag{17-5}
\end{align*}
$$

The expression for K , gases enter as their partial pressures in atmospheres and ions or molecules in solution as their molarities (M).

Ex. 17-8: Using $\Delta G_{f}^{o}$ tables in Appendix 1, calculate the solubility product constant $\mathrm{K}_{\text {sp }}$ for $\mathrm{PbCl}_{2}$ at $25^{\circ} \mathrm{C}$.
Ans:

$$
\begin{aligned}
& P b C l_{2}(s) \longrightarrow \mathrm{Pb}^{2+}(a q)+2 \mathrm{Cl}^{-}(a q) \\
& \Delta G^{o}=\left[1 \cdot \Delta \mathrm{G}_{f, \mathrm{~Pb}^{2+}(a q)}^{o}+2 \cdot \Delta \mathrm{G}_{f, \mathrm{Cl}^{-}(a q)}^{o}\right]-\left(1 \cdot \Delta \mathrm{G}_{f, \mathrm{Pbll}_{2}(s)}^{o}\right) \\
&=[1 \cdot(-24.4)+2 \cdot(-131.2)]-[1 \cdot(-314.1)] \\
&=+27.3 \mathrm{~kJ} \\
& \Delta G^{o}=-\mathrm{RTlnK} \\
& 27.3=-8.31 \times 10^{-3} \cdot 298 \cdot \ln \frac{\left[\mathrm{~Pb}^{2+}\right] \cdot\left[\mathrm{Cl}^{-}\right]^{2}}{1} \\
& 27.3=-8.31 \times 10^{-3} \cdot 298 \cdot \ln \mathrm{~K}_{s p} \\
& \ln \mathrm{~K}_{s p}=\frac{27.3}{-2.42}=-11.0 \\
& \mathrm{~K}_{s p}=\mathrm{e}^{-11.0}=-1.67 \times 10^{-5}
\end{aligned}
$$

The relationship $\Delta G^{o}=-\mathrm{RTln} K$ allows us to relate the standard free energy change to the extent of reaction. Consider:

$$
A(g) \rightleftarrows \mathrm{B}(\mathrm{~g}) \quad \Delta \mathrm{G}^{o}=20 \mathrm{~kJ}
$$



Fig. 17.8: Dependence of extent of reaction on the value of $\Delta G^{o}$ for the general system $A(g) \rightleftarrows \mathrm{B}(\mathrm{g})$.
$\mathrm{g} \Delta \mathrm{G}^{o} \approx 20 \mathrm{~kJ}$, the reaction does not occur $\Rightarrow 100 \% \mathrm{~A}$
gif $\Delta \mathrm{G}^{\circ} \leq-20 \mathrm{~kJ}$, the equilibrium constant is so large that virtually all of A converted to $B \Rightarrow 100 \% B$.
gif $-20 \mathrm{~kJ}<\Delta \mathrm{G}^{o}<20 \mathrm{~kJ}$ will the equilibrium mixture contain appreciable amount of both A and B .
gif $\Delta \mathrm{G}^{\circ}=0, \mathrm{~K}=1 \Rightarrow$ equal amount A and B .

## § 17-7 Additivity of Free Energy Change; Coupled Reaction

Free energy changes for reactions, like enthalpy or entropy, are additive. That is:

If
Reaction $3=$ Reaction $1+$ Reaction 2
Then

$$
\Delta G_{3}=\Delta G_{1}+\Delta G_{2}
$$

Metabolism（新陳代謝）reaction：

$$
C_{6} H_{12} O_{6}(a q)+6 \mathrm{O}_{2}(g) \longrightarrow 6 \mathrm{CO}_{2}(g)+6 \mathrm{H}_{2} \mathrm{O} \quad \Delta \mathrm{G}^{o}=-2870 \mathrm{~kJ}
$$

ordinarily do not serve directly as a source of energy．
Instead these reactions are use to bring about a nonspontaneous reaction：

$$
A D P(a q)+\mathrm{HPO}_{4}{ }^{2-}(a q)+2 \mathrm{H}^{+}(a q) \longrightarrow \mathrm{ATP}(\mathrm{aq})+\mathrm{H}_{2} \mathrm{O} \quad \Delta \mathrm{G}^{o}=+31 \mathrm{~kJ}
$$

腺苷二磷酸
腺苷三磷酸

In the coupled reaction with glucose，about 38 mole of ATP are synthesized for every one mole of glucose consumed．

$$
\Delta G^{o}=-2870+38 \cdot(+31)=-1692 \mathrm{~kJ}
$$

Ex．17．9：The lactic acid $\left(C_{3} H_{6} O_{3}(a q), \Delta \mathrm{G}^{o}=-559 \mathrm{~kJ}\right)$ produced in muscle cells by vigorous exercise eventually is absorbed into the bloodstream， where it is metabolized back to glucose $\left(\Delta G_{f}^{o}=-919 \mathrm{~kJ}\right)$ in the liver． The reaction is：

$$
2 \mathrm{C}_{3} \mathrm{H}_{6} \mathrm{O}_{3}(\mathrm{aq}) \longrightarrow \mathrm{C}_{6} \mathrm{H}_{12} \mathrm{O}_{6}(\mathrm{aq})
$$

a）．Calculate $\Delta G^{o}$ for this reaction，using free energies of formation．
b）．If the hydrolysis of ATP to ADP is coupled with this reaction， how many moles of ATP must react to make the process spontaneous？
Ans：
a）．$\Delta G^{o}=\left[1 \cdot \Delta \mathrm{G}_{f, \mathrm{C}_{6} H_{12} \mathrm{O}_{6}(a q)}^{o}\right]-\left[2 \cdot \Delta \mathrm{G}_{f, \mathrm{C}_{3} H_{6} \mathrm{O}_{3}(a q)}^{o}\right]$

$$
=[1 \cdot(-919)]-[2 \cdot(-559)]
$$

$$
=+199 \mathrm{~kJ}
$$

b）．$\Delta G^{o}<0$
$+199+x \cdot(-31.0)<0$
$\mathrm{x}>6.4 \mathrm{~mol}$ ATP
$\Rightarrow \uparrow \S \Downarrow \rightarrow \therefore \sim \sim$

